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ABSTRACT

One of the strengths of the Pennsylvania Council of Teachers of Mathematics (PCTM) is that it gives mathematicians and mathematics educators the opportunity to exchange and contribute to each other's professional growth. The topic for this 1985 yearbook, the first of a proposed ongoing series, has been chosen to coincide with the annual PCTM meeting. This yearbook contains 10 articles related to calculators, computers, and problem solving. Discussions include: (1) using computers in problem solving; (2) using LOGO; (3) using computers with the intermediate grades for problem solving; (4) heuristic approaches in using computers; (5) calculators in the classroom; (6) number operations; (7) math anxiety; (8) mathematics achievement in Pennsylvania; (9) the participation of women in computers and mathematics; and (10) classroom applications. (CW)

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1985 Yearbook

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MATHEMATICS EDUCATION AT THE MIDPOINT OF THE AGENDA

1985 Yearbook

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Preface

The 1985 Yearbook is the first to be developed under a new concept recommended by the Executive Board of the Pennsylvania Council of Teachers of Mathematics. The guidelines specify that a yearbook addressing a theme of timely interest to the wide range of PCTM membership be prepared on a schedule that would allow a new one to be presented at each annual meeting.

The theme for the 1985 PCTM Yearbook is "1985: Mathematics Education at the Midpoint of the Agenda." Acceptable topics included — but were not limited to — Calculators, Computers, and Problem Solving.

The successful completion of this yearbook is a direct result of the dedication and talents of the authors of these essays. They worked hard to produce papers of high quality and adjusted personal schedules to meet production guidelines. We owe them our sincere gratitude. The ultimate reward, however, to all those who have contributed to this yearbook will be the extent to which the ideas contained herein result in improving the teaching and learning of mathematics.

We are glad to have had the opportunity to serve as editors during this inaugural effort. It is our hope that this yearbook concept is now firmly established and that it will continue to grow in stature.

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1985 Yearbook Editors

COMPUTERS FOR PROBLEM SOLVING: MIDPOINT OF THE 80s

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Progress through the NCTM Agenda for the 80s mirrors that of a successful problem solver through the unmapped solution territory of an elusive problem. At times the strides on particular Agenda items have been necessarily halted, waiting for the other pieces of the Agenda to catch up. With surprising frequency, however, progress on the Agenda has been sudden and powerful, far outdistancing the imaginations and ambitions of those who have presumed to lead the way. At those times, Agenda goals have merged, producing a singularly promising and harmonious picture of the direction for school mathematics. At the midpoint of the 1980s several Agenda goals have met and offer unprecedented opportunity for powerful headway toward fulfillment of the spirit of the NCTM's Agenda.

Two Agenda items which now offer the striking promise of unification of purpose are those centering on problem solving and on the use of computer devices. In 1985, the nation stands ready for renewed efforts in mathematics education and in the integration of computers in the schools. Recent national reports have provided consistent reminders of present deficiencies in the mathematics backgrounds of the nation's youth. Government funding is beginning to target the improvement of mathematics education as a goal. At the same time national attention had turned to the betterment of what goes on within the walls of the mathematics classroom, rapid progress in the field of microcomputing has placed powerful computing devices at the mathematics classroom door. Daily TV commercials and ubiquitous magazine ads admonish parents that a child's success in school is inextricably linked to the family purchase of a home computer. The public wants more mathematics education and it is being told that that education should be computer-enhanced.

One national report that has, perhaps, captured more attention in education circles than in the public arena is the National Assessment of Educational Progress. Although the public may have been generally pleased with the NAEP good news that the nation's youth had improved in computational skills, mathematics educators could not help but note the disparaging news that students were woefully inadequate at solving anything more than a one-step problem. More recent research has driven home the powerful message that even college students successfully complete mathematics courses with little ability to use the processes that

they have mastered (Clement, 1982). Not only do the nation's youth need more mathematics, but the nature of mathematics instruction needs careful reassessment.

The classroom computer of 1985 offers unparalleled opportunity for needed reform of school mathematics. Recent cries for excellence in education beg for enhancement of students' higher level thinking, and never before have affordable computers been so capable of assisting in this task. The opportunity is a new one, shored up by a newly popular vision of the computer as an aid to problem solving. When minicomputers first came within the financial grasp of school budgets, educators were entering the "back to basics" movement and seized on computers as potential classroom aids for patient tutoring and unfailing drill and practice. Initial time and effort concentrated on developing tutorial and drill-and-practice software for lower level skills. Available output devices were confined mainly to line printers with limiting serial processing. The graphics capabilities needed for enhancement of concepts and of higher level thinking was simply not available in the school. The computer uses of choice were to teach students to program or to test them on acquisition of lower level skills. The advent of the microcomputer with its astonishing graphic, numeric, and symbolic capability has opened new worlds to school mathematics computer applications. The early 80s have seen intriguing experimentation in using the computer to enhance problem solving, to develop mathematical concepts, and to encourage higher level thinking.

To analyze the potential of the microcomputer in the quest for a deeper approach to secondary mathematics, two perspectives offer newly fertile ground for exploration. First, what is the current thought on the nature of thinking skills which ought to be part of the secondary mathematics student's repertoire? Second, at what types of higher level processes can microcomputer mathematics lessons be addressed?

WHAT TYPES OF THINKING SKILLS SHOULD BE TAUGHT?

The development of thinking ability is a major if unspoken goal of mathematics education. Yet students do not seem to acquire this ability as a natural outcome of their encounters with the mathematics curriculum. How can the mathematics curriculum be designed to enhance analytical ability? What facets of thinking skills ought to be taught?

Answers to these questions are as elusive as the formation of the questions themselves. Raymond Nickerson recently described a categorization of programs to teach thinking skills in the classroom (Nickerson, 1984). The categorization he describes provides a framework within which possible answers will be placed. Nickerson, Perkins, and Smith (Nickerson, Perkins, and Smith, in press) placed thinking skills programs into five categories:-

"(1) cognitive-process approaches,
(2) heuristics-oriented approaches,
(3) approaches that focus on the development of formal thinking in the Piagetian sense,
(4)approaches that emphasize language and symbol manipulation, and
(5)approaches that focus on thinking as subject matter."
(Nickerson, 1984, p. 29)

Goals for mathematical thinking espoused by mathematics educators fall into each of these categories. The discussion which follows will focus only on the first two categories.

Cognitive Processes

Increasing interest in enhancing thinking ability in students has found one of its homes in the newly developing field of cognitive process instruction. Cognitive process instruction is based on the premise that thinking skills can be isolated and taught. Falling into the domain of cognitive processes are many of the skills which receive attention in secondary school instruction for mathematical problem solving: making logical inferences, finding patterns, sequencing computations, classifying mathematical objects. College instructors seem to assume student mastery of still other cognitive processes (Arons, 1979). Many of the processes cited by Arons translate readily to mathematics students. College mathematics students, for example, are expected to recognize and manipulate meaningfully variable quantities in verbal problem contexts. They are expected to be comfortably adept at proportional, combinatorial, correlational, and probabilistic reasoning. They should be able to formulate and understand syllogistic reasoning. They should be able to reason accurately in a closed system, carefully noting and using underlying hypotheses. They should be able to identify gaps in information. They should be able to verify the conclusions and interpret their results in applied contexts. Among their cognitive skills should be the ability to check for internal consistency, the ability to construct several paths to a problem's solution, and the ability to note the importance of special cases.

That students who understand mathematics possess finely tuned versions of the aforementioned skills seems undeniable. As has been pointed out by Krutetskii in a longitudinal study of mathematically talented Soviet youth, those who do well in mathematical problem solving demonstrate an unshakeable understanding for the structure of the problems they solve (Krutetskii, 1976). This structural understanding is evidenced by student abilities in many of the thinking skills mentioned above. Very able mathematics students are distinguished by their abilities:

(1) to know when needed information is missing from a problem's statement,

(2) to recognize redundant or superfluous information almost instananeously,
(3) to reason automatically about the most general case of a problem,
(4) to produce several different paths to a problem's solution, one immediately following the other,
(5) to switch freely from a direct to an indirect train of thought. (Krutetskii, 1976).

The correlation between abilities shown by Krutetskii's very able elementary and secondary students and the abilities college instructors expect is striking. The aforementioned cognitive processes seem to be ones which mathematically talented students possess and which college instructors expect in their students. It remains to be seen, of course, whether these processes or the structural understanding or problems can be taught. Furthermore, once the processes or structure are taught, it is still unclear whether the new knowledge will transfer appropriately to problem solving. If cognitive process instruction is a viable route to successful problem solving, however, the list generated above seems to be a defensible one.

Heuristics

Just as mathematics educators have directed considerable attention to teaching specific thinking skills, they have also focused on improving their students' problem solving strategies or heuristics. Some attempts to teach heuristics have centered on general strategies for problem solving. Such programs typically represent some variation of George Polya's four-step theme for problem solving: understand the problem, plan a solution, carry out the plan, and examine the solution obtained (Polya, 1957). Other programs focus on more specific heuristics such as: construct a diagram, use successive approximations, think of a related problem, restate the problem, examine special cases, construct a table, look at a simpler case, select and pursue subgoals, or divide the problem into cases. There is some research evidence that when students are taught heuristics, they use the strategies with greater frequency and achieve correct solutions more often (Suydam, 1980). If problem solving, is, as the Agenda recommends, to be the focus of school mathematics, attention seems properly directed to on teaching heuristics.

HOW THE COMPUTER CAN BE USED TO ENHANCE PROBLEM SOLVING

The early 80s have witnessed widespread experimentation with uses of the computer in the mathematics classroom. Although a considerable portion of that research has been concerned with constructing better classroom diagnostic and managerial applications, a significant number of recent projects have been premised on efforts to enhance student understanding of mathematical concepts and processes. These latter

efforts fall largely into three categories of computer use:

(1) The computer is used to provide students with practice on particular thinking skills.
(2) The computer is used to increase student understanding of particular mathematics concepts.
(3) The computer is used to give students increased opportunity for development of problem solving strategies.

The first two categories correspond to the previously discussed vision of cognitive process instruction. Practice on thinking skills, the first category, is the modus operandi of cognitive process instruction. Such instruction is premised on the notion that thinking skills can and should be taught. The second category, development of conceptual understanding, responds to Krutetskii's findings that more able students better understood the structure of mathematical concepts. Although the direction (or existence) of causality in Krutetskii's study is undetermined, it is conceivable that better conceptual understanding could result in better problem solving. Computer efforts to increase understanding of particular mathematics concepts could be viewed as attempts to enhance problem solving. The third category encompasses computer-based attempts to develop heuristics.

This section will describe a range of ways in which the computer has been used to enhance problem solving, and conclude with some observations about the future promise of computer use directed at higher order thinking.

USE OF THE COMPUTER TO IMPROVE COGNITIVE PROCESSES

Claims that the computer can help improve students' thinking processes are occurring with increasing frequency and growing conviction. Two types of arguments underlie these claims: first, the computer improves student thinking since it provides students with the general opportunity to think; second, the computer enhances student thinking because it provides opportunity for practice on particular thinking skills.

Many of those who extol the virtues of the computer as a tool for thinking focus on programming applications. A number of educators have explored the potential of Logo for allowing students to discover mathematical concepts (Billstein, 1983). Literature is abundant with examples of elementary school students freely manipulating the Logo turtle in ways which seem to reflect an understanding of the mathematical concepts of angle measure, symmetry, and variables. Others have explored the use of student programming in more traditional languages to enhance thinking skills. Luehrman, for example, has long argued that programming skills are an essential component of computer literacy (Luehrman, 1981). Some embed programming in related exercises. Elgarten, for example, outlines a sequence of programming-related activities

which he proposes to precede student use of programming as a problem solving tool. Before his students write a computer program they engage in a four-step analytic process:

"(1) Write an analysis that includes the development of an algorithm;
(2) write a plan for a computer solution to the problem;
(3) develop a flowchart;
(4) code a program to solve the problem and run it on the computer."
(Elgarten, 1984, p. 459).

This implementation of programming would seem to provide students with practice in cognitive skills of analysis and sequencing as well as draw student attention to a general heuristics.

The role of the computer in the practice of particular thinking skills has drawn considerable recent attention. The list of skills claimed to be enhanced by various computer uses includes many of the thinking skills which recent educational programs have targetted. The computer has special capacity for providing students with practice on mathematical thinking skills in three areas:

(1) organizing and seeing patterns in data,
(2) making and checking conjectures about quantitative relationships,
(3) investigating alternative interpretations and meanings for mathematical concepts.

Examples of ways in which educators have used the computer to practice these skills are discussed below.

Organizing and Seeing Patterns in Data

The search for patterns has long dominated the study of mathematics and has historically depended on long hours of patient work in data generation and handling. The unusual capacity of the computer for generating, storing, retrieving, and rearranging data, places this search for patterns within the grasp of a much larger population. Computer educators have taken decided advantage of this expansion of information handling capacity within the context of mathematics classes. One rendition of this use is for students to search for a relationship between the equation for a function and its graph (Hatfield, 1983; Koslov and Ott, 1984). Students vary a designated parameter on equations such as $y = a\,x + b$ or $y = a \cos (b\,x)$ and examine the effect of these changes on a computer-generated graph. Other uses of the computer as data generator center on algebraic and numeric results. Winter, for example, used the computer to generate positive integral values of a, b, and c, which satisfied $\frac{1}{a} + \frac{1}{b} = \frac{1}{c}$. Once students could examine sets of results, generalizations like

$$\frac{1}{2n} + \frac{1}{2n} = \frac{1}{n} \qquad \text{and} \qquad \frac{1}{4n} + \frac{1}{12n} = \frac{1}{3n}$$

would abound (Winter, 1984). Programs like TK! Solver and Visicalc offer

new opportunities for student practice with the organization and pattern recognition of large sets of data. When sophisticated computer programs are used to generate and organize, students can concentrate on making and testing conjectures.

Making and Testing Conjectures

Computer generated data sets can not only lead students to conjectures about graphical and algebraic relationships but can actually suggest viable methods of proof. A striking example resulted from Hay's work on a favorite geometry problem. When first posed to this writer, it read something like this:

> Start with triangle ABC. From each of the vertices move equal distances counterclockwise along adjacent sides. Mark the points on the sides where you stop as D, E, and F. If Δ D E F is equilateral, show that Δ A B C must also be equilateral.

The problem is one which has contributed hours of problem solving pleasure to geometry students and teachers alike, and most often those hours have been uninterrupted by the closure brought on by success. Hay's computer-generated problem data led her not only to the discovery of an important but overlooked fact but also to the discovery of a successful proof strategy (Hay, 1981).

With easy computer-generation of data, both numeric and graphical, students have access to a broader base for conjecture. With a broadening array of mathematics utility programs and with more widespread knowledge of programming, students are more able than ever to check their conjectures. This increased access spans the range of ages and mathematical levels. With a computer program which generates area representatives of given fractions, for example, students might derive a notion of equivalent fractions (Hatfield, 1983). Using the same programs, they could easily test their conjectures. With computer help in generating and testing conjectures, Calculus classes have become places for students to test "What if . . ." questions about changes in problem conditions and parameter values (Smith, 1982; Heid, 1984).

Investigating alternative representations

Foundational to the understanding of mathematics is the ability to conceptualize quantitative relationships in a variety of representations. The computer opens new worlds of representations and interpretations to its user. With appropriate programs, students can view compound interest problems not just a search for a single value but as the examination of an array of growing values. In finite mathematics classes at University of Maryland, for example, students solved a variety of business application problems by examining tables of growing present values. As Luehrman points out, programming itself provides students with

another means of representing a problem (Luehrman, 1981).

With the expanding special capabilities of today's classroom computers, mathematics instructors can provide students with practice on particular thinking skills. They can exploit the data-processing capacity of the computer to give students experience in examining arrangements of data, in making conjectures about the quantitative relationships involved, and in testing those conjectures. They can use the graphical, numerical, and symbol-processing capacities of the computer to generate opportunities for students to create and explore new representations for mathematical concepts. Prototypes, like those discussed above, are available for the use of the computer as a tool to promote thinking. The territory has not yet been fully explored. One could accept the challenge of using the computer to enhance thinking skills with both the inspiration of previously successful attempts and the allurement of opportunities to explore unmapped terrain.

USE OF THE COMPUTER TO INCREASE CONCEPTUAL UNDERSTANDING

Increased conceptual understanding may be an essential key to improved problem solving. The computer has important potential for the development of that understanding. Present and projected computer power provides daily new opportunities for mathematics teachers to use the computer in improving student understanding on course concepts. Present computer applications aimed at increasing conceptual understanding take three major forms:

(1) Concepts which were previously inaccessible can now be taught;
(2) Traditional concepts can be represented in new ways;
(3) Conceptual understanding can assume a newly important and fundamental role.

New Concepts

Educators have used the power of the computer to teach concepts whose coverage was never before within the reasonable grasp of school mathematics students. Just some of the new concepts which have been taught in computer-enriched classrooms are drawn from number theory, probability, infinite series, and theory of equations. Examples appear frequently in math education literature. Donahue, for example, describes how students can use the computer to study a number theory topic, the generation of palindromic sums. A palindrome is a number which reads the same forwards and backwards, like 121. When the sum of number and its reverse produces a palindrome, the sum is "palindromic." (Donahue, 1984). Probability notions are also more accessible when a computer is used. Probability ideas can be taught using Monte Carlo programs (Hill, 1983). The notion of randomness comes within the grasp of

fourth and fifth graders when it is taught within the context of computer games (Winner and McClung, 1981). Infinite series has been another fertile topic. Applications of infinite series (Koetke, 1983) are suggested for computer-able classes. Even complicated algebraic study of infinite series (e.g., $\sum \frac{1}{n^p}$) becomes possible with the use of symbol manipulation programs such as muMath (Kunkle and Burch, 1984). Once discarded, theory of equations takes on new importance when stripped of its computational complexity. With appropriate graphing programs, secondary students can now study the behavior of higher degree polynomials (Kennedy, 1981). The list of possible new topics seems endless.

New representations

Not only can new concepts be taught, but traditional concepts can take on new meaning with creative use of computer tools. For young children, Logo engages them in experiences with concrete embodiments of direction motion, and order relationships among numbers (Nelson, 1981). Rudimentary BASIC programs give them hands-on-experience with the notion of a variable (Shumway, 1983). Simple BASIC programs offer alternative representations for the relationships between repeated addition and multiplication (Wilson, 1984). Older elementary school students beginning the study of rational numbers obtain a richer conceptualization when they experience computer-generated representations of fractions as "stretchers" and "shrinkers" (Hatfield, 1983). Students who have previously considered the properties of quadratic relationships only in compartmentalized fashion can begin to see relationships among them through dynamic computer displays. Kimberling, for example, constructed two programs which "enable students to simulate the 'explosion' of ellipses into parabolas and then into hyperbolas." (Kimberling, 1984, p. 365). The notion of the derivative function also takes on new meaning for Calculus students who use the computer to investigate the shape and properties of a slope function (Lappan and Winter, 1982; Heid, 1984).

The computer can thus be used to avail students at all levels to heretofore unaccessed representations of the concepts they study. Computer-generated displays of relationships among concepts have the potential for a visual impact which far transcends the usual symbol-based arguments for those relationships. Even students who have studied the notion of eccentricities of quadratic relations will derive still deeper understanding from a dynamic graphical representation of changing eccentricity. Likewise, Calculus students who have mastered an entire range of derivative formulas will still find it informative to investigate relationships among simultaneously displayed graphs of a function and its first few derivative functions. In providing student access to new representations of concepts

and of relationships among concepts, the computer opens the door to a richer understanding of mathematics.

New role

The power of the microcomputer for routine calculation, for execution of established algorithms, for generation of graphics, and for symbolic manipulation, poses a question of undeniable importance for mathematics educators. If educators tap fully the capacity of the computer so that students can expect universal access to powerful "tool" programs, can school mathematics programs begin to shift attention from mastery of routine algorithms to development of mathematical concepts? Although essential aspects of this question are yet to be formulated, recent work has shed interesting light on how and whether such computer-based conceptual development is possible.

Statistical and probabilistic concepts have been popular targets for using the computer for routine procedures to aid in the development of concepts. The choice is a natural one since the symbol manipulations underlying these concepts are complicated and possibly distracting if done by hand. Educators have proposed and outlined ways to use computer-generated results to provide incisive demonstrations of the central limit theorem (Thomas, 1984), of hypothesis testing, random sampling, probability distributions, statistical inference, and p-values (Morgan and Morgan, 1984), and a large variety of other fundamental statistical notions (Simon, Atkinson, and Shevokas, 1976; Travers and Gray, 1981). If the computer is used to perform the calculations, students would conceivably develop an intuitive grasp of concepts in a minimum of instructional time.

The computer can also be used to process algorithms and shift initial emphasis to conceptual understanding in the study of beginning calculus. The computer can execute all the usual calculus algorithms while students concentrate on interpreting and constructing conceptual representations. Since the computer provides easy access to concrete examples, this approach contains an important dimension absent in pre-computer conceptually oriented curricula. In a recent classroom experiment, students exposed to this conceptually-oriented calculus seemed to learn Calculus notions more deeply without noticeably detrimental effects on their facility with routine algorithms (Heid, 1984). The impressive power of the computer conjures up images of a transfigured mathematics curriculum which successfully develops sound notions of mathematical concepts.

USE OF THE COMPUTER FOR DEVELOPMENT OF HEURISTICS

When mathematics educators have concentrated efforts on using the computer for the enhancement of problem solving strategies, a variety of

tactics have emerged. Some have directed attention to constructing activities which focus student attention on Polya's four problem solving steps. Channell, for example, designed problem solving activities which led students step-by-step through understanding and analyzing problems before he presented computer programs for them to use in generating solutions. His problems (on maximizing the volume of a box of fixed surface area and on computing the expected number of trials needed to achieve a goal) lent themselves well to computer analysis by inspection of tables. (Channell, 1984). The computer being a natural table generator, students could concentrate on understanding, analysis, and interpretation. Kantowski has done perhaps the most extensive work with teaching problem solving strategies using the computer. Her work on "The Microcomputer and Problem Solving Processes in Middle School Mathematics" (NSF Project #SED 8016561) resulted in the creation of a series of microcomputer programs focused on developing student facility with each of Polya's stages. The programs present the student with a problem and offer options for help in a variety of contexts. Students can ask the computer for a diagram, a picture, or a table, to illustrate the meaning of a problem. They can ask for a suggested strategy, an appropriate computer programs, a related formula, or an answer to the problem (Kantowski, 1983). The computer, used in this fashion, can focus student attention directly on a particular strategy, making students more conscious of the need and usefulness of those strategies.

While Channell and Kantowski centered explicit instruction on general heuristics, others have designed activities aimed at encouraging student creation of their own strategies. One of the first important efforts in this direction was Dugdale's "Green Globs." This computer posed to students the problem of creating equations for relations which would pass through thirteen randomly generated points. The game format of the program encouraged students to compete in finding the most elegant solution for a given round. The result was student discovery of surprisingly sophisticated equation strategies (Dugdale, 1982). Whereas Dugdale's approach was premised on a carefully constructed environment of mathematical symbolizations, Inman and Clyde's work suggests the potential that real-life applications might have in encouraging students to create their own strategies. (Inman and Clyde, 1981). With the computer as a tool and with realistic and rich problems as a starting point, students might feel a natural compulsion to use the computer in their investigations of aspects of the problem.

CONCLUSIONS

At the midpoint of the 80s, one finds a strikingly good match between the list of thinking skills educators name as important goals and the ways in which mathematics educators are proposing the computer be used.

Impressive progress has been made in the last few years in the design of ways the computer can give students practice with thinking skills and problem solving strategies. Mathematics educators have proposed myriad new computer applications which make thoughtful use of computing tools (Fey, 1984). The potential for computer-based development of mathematical concepts is extensive, and growing. In 1985 the promise of the computer for development of higher order thinking is considerable. Whether this marriage of computers and higher order thinking will last through the 80s is, of course, an open question — whose answer will depend largely on the recognition and exploitation of this potential by classroom teachers.

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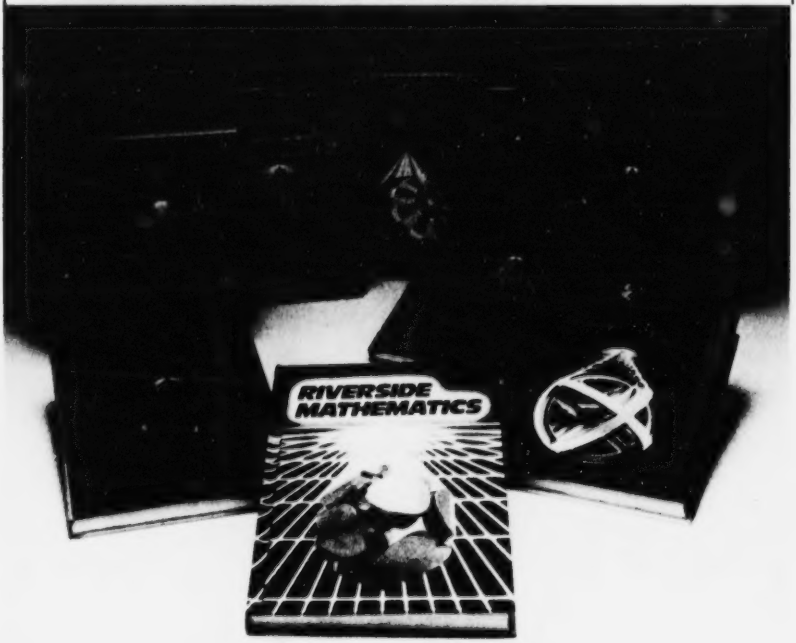

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IS LOGO MORE THAN FUN?

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LOGO lures students into learning by exploration and discovery — and they love it. There is material inherent in LOGO that students easily form into learning models to which they can relate new ideas or concepts. However, there are still many complex questions to be answered about LOGO's use as a teaching tool. (Nelson, 1981, p. 16)

Educators working with LOGO will agree that it is a highly motivating and exciting computer language that their students enjoy. Students will arrive early and stay late for classes if guaranteed the time to explore and develop procedures in LOGO.

In early experiments with LOGO and elementary students, Feurzeig (1972) discovered that children were delighted to use computers with LOGO to express their ideas in original personal styles. A more recent study during the summer of 1981 at the Microcomputer Resource Center at Teachers College, Columbia University introduced children to computer literacy and programming using the language LOGO, while exploring various instructional techniques to teaching it. The results of the project definitely demonstrated that LOGO can be appealing to students of all ages and abilities. Students evinced much diversity in their accomplishments and expressed satisfaction with the experience in a variety of ways. (Billings, K.; Dunne, J.; Simpson, G.; Spielvogel, B.; & Wolz, U.; 1981)

Although LOGO was developed nearly twenty years ago, it is the "buzz word" in elementary computer education today. The question to be answered is: "Is LOGO more than fun?" The significant educational value of LOGO resides in the interactive relationship between learners and this powerful computer language. Educators have much to learn about the learning process and about individual differences in learning styles while using LOGO.

Seymour Papert, the "father" of LOGO, contends that even the simplest LOGO exposure can awaken new opportunities for sharpening one's thinking. "Programming the LOGO TURTLE starts by making one reflect on how one does oneself what one would like the TURTLE to do." (Papert, 1980, p. 28) This type of experience with computers can lead one to reflect on one's own actions and thinking, thus helping to develop more reflective self-conscious thinkers. Research has shown that reflective thinkers can improve their performance on visual discrimination tasks, serial recall, inductive reasoning, reading, and problem-solving

skills. (Kagan, Pearson, & Welch, 1966). An exercise in LOGO "can lead to greater confidence about the ability to choose a cognitive style that suits the problem." (Papert, 1980, p. 27)

Papert's statements concerning reflective thinking and cognitive style led this researcher to investigate the effect of the LOGO computer programming environment upon the reflective and impulsive cognitive styles of second-grade students. (Young, 1982)

Second-grade students for this study were initially identified as reflective or impulsive by their scores on the Matching Familiar Figures Test: Child. An experimental group of twelve impulsive students was then placed in the LOGO computer programming environment for fifteen one-hour interaction periods daily. These interactions were closely observed by the researcher. Behavioral aspects of reflective/impulsive strategies and specified instructional objectives were recorded by a systematic procedure for daily identifying and categorizing developed especially for the study.

Control groups of twelve students initially identified as impulsive and twelve initially identified as reflective, experienced no interaction with LOGO or any other computer programming environment.

At the conclusion of the treatment, both experimental and control groups were post-tested on the Matching Familiar Figures Test: Child. The results of the post-test and two other researcher-designed instruments, individual Reflective-Impulsive Strategies Checklist and individual Instructional Objectives Checklist, were used to analyze and summarize the data.

The results of the study were:

1. No statistically significant relationship exists between the impulsive experimental and the impulsive control groups and the final outcome of the impulsive/reflective score on the MFFT: Child posttest due to placement in a LOGO computer programming environment.
2. More of the students in the impulsive experimental group shifted in the direction of reflective thinking than did those students in the impulsive control group.
3. None of the twelve students initially identified as impulsive on the MFFT: Child pretest scored reflective as a result of a daily three-week interaction with the LOGO computer programming environment.
4. The individual checklists recording daily usage of reflective/impulsive strategies by the experimental group indicated that all students were using more reflective strategies at the conclusion of the LOGO treatment than they were at the outset.
5. Six of the twelve students in the impulsive control group remained impulsive following a period of no interaction with the LOGO environment.

6. Eight of the twelve students in the reflective control group remained reflective following a period of no interaction with the LOGO environment.

Other findings of the study were:

1. The checklist recording daily usage of reflective and impulsive strategies by the experimental group indicated that those students who scored unarticulated on the posttest used more reflective strategies than those who scored impulsive.
2. The daily checklist measuring attainment of mastery of the instructional objectives demonstrates that students need differing amounts of time in their individual paths to learning and discovering the elements of the LOGO computer programming language.

In conclusion, the results of this study indicate that a three-week placement in the LOGO computer programming environment is not sufficient to change the dichotomous classification of impulsive on the MFFT: Child to reflective. However, a scrutiny of the increased use of reflective strategies and the shift toward reflective that eight of the twelve students in the experimental group experienced, does suggest that a longitudinal exposure to LOGO might cause impulsive thinkers to become reflective.

Other conclusions gleaned from this study support the findings of the pioneer teaching experiments with LOGO. Although the LOGO computer programming environment experience did not significantly alter the reflective/impulsive cognitive style of the students in the experimental group, the experience permitted them to find their own level of operation in computer programming. They all were able to design computer programs while developing self-confidence in their abilities and pride in their accomplishments.

LOGO has been used with children of all ages and ranges of abilities. It promotes a close association between a child's cognitive development and an environment where he/she takes the active role in learning. (Weir, 1981) It represents a fundamental change in educational concepts and reflects effects the computer can make in society as a whole. (McCorduck, 1979) A general theme from all LOGO projects is that LOGO can be effective for all students in a school setting. (Watt, 1982) Although Seymour Papert has many intriguing ideas on revolutionizing education with microcomputers and much research has been conducted with LOGO, much more remains to be done. Future research needs to explore further the relationships of LOGO to particular cognitive styles, various learning theories, problem solving techniques, and the role of the teacher and the entire educational system.

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COMPUTERS AND PROBLEM SOLVING IN THE INTERMEDIATE GRADES: THEY BELONG TOGETHER

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As mathematics educators move toward implementing the NCTM's recommendations for school mathematics in the 1980's, opportunities should not be overlooked to integrate the development of problem solving skills (recommendation #1) and to take advantage of the power of calculators and computers (recommendation #3). By the time children reach the intermediate grades (grades 5 through 8), they should be beginning to develop a repertoire of identifiable problem solving strategies such as "draw a picture," "guess and check," "look for a pattern," and "list all possibilities."

Concurrently, children at this stage should also be developing the ability to recognize problem situations where some specific strategies are appropriate to use and some are not. Consider the "Nine Dot Problem" in which you are asked to draw four straight line segments through nine dots (see figure 1) without lifting your pencil from the paper.

Figure 1

This is not a problem for which a computer would be an appropriate tool, at least at the elementary-middle school level. The solution usually requires a strategy of "changing your point of view" as the problem solver plays with the problem and suddenly realizes that the line segments don't have to stay within the dot array to achieve a solution.

Problems involving hidden assumptions such as the "Missing Dollar Problem" aren't usually thought of as algorithmic or "computer type" problems either. In the "Missing Dollar Problem," three men share a hotel room and pay $30 for it. The desk clerk discovers that he should have charged them only $25 and sends the bellhop up to the room with a $5 refund. The bellhop can't divide $5 evenly by 3, so he returns only $3 to the men and keeps $2. Therefore, each man paid only $9 as his share of the room for a total of $27, with the bellhop keeping the other $2 for a total

of $29. What happened to the extra $1?

Of course, the solution depends upon not falling into the false assumption that the total room charge to the men plus the bellhop's tip should equal $30. Solving this problem does not depend on utilizing a computer's ability to grind out long, often recursive, computations. As in the Nine Dot Problem, a computer would be of little value in solving the Missing Dollar Problem.

Children should be exposed to a wide variety of problem types which hone their abilities to select appropriate problem solving strategies. Thus, it is the teacher's role to provide a learning environment which leads students to learn to choose appropriate problem solving strategies as well as to utilize appropriate tools such as models, drawings, tables, graphs, calculators, and — the focus of this article — computers.

Let me describe a sequence of activities that will illustrate how children can employ a computer as an appropriate problem solving tool. These occurred at our campus school (grades K-6) during the past year. I was working with a group of ten and eleven year olds exploring computer related topics, now a regular part of our campus school's curriculum.

The children had already learned to write programs in the language BASIC using commands such as "LET," "PRINT," "INPUT," and "READ-DATA." Also, they had worked with simple programs that use "IF-THEN," "GO TO," and "FOR-NEXT" commands. I had previously used Cuisenaire Rods to explore the relation of the length of each rod in centimeters (the number of white rods written in the □) and the surface area in square centimeters for each of the ten basic rods (written in the △). A table was made to summarize the relation.

□	1	2	3	• • •	10
△	6	10	14	• • •	42

Figure 2

As the table was being constructed, I asked the children to predict the pairs in the table after we had actually checked the first four rods. They were able to predict that as each number in the □ row increased by one, the number in the △ row increased by four (look for a pattern). I then asked them if they could write a program in BASIC that would generate the □ and △ values for any train of white rods up to 100. With some "guess and checking" a workable program was developed (figure 3) and printed.[1]

[1]Note: All programs listed in this article were written in Applesoft II BASIC using a different line number for each command. In order to conserve page space, I have translated the programs into a format using multiple commands on one numbered line.

```
10 LET X = 6
20 FOR N = 1 TO 100: PRINT N, X : LET X = X + 4 : NEXT N
30 END
```

Figure 3

With older or gifted children, I would have extended this activity to include guessing a formula that would calculate the surface for any train of white rods (i.e., $\triangle = 4 \times \square + 2$). The program in figure 3 could then be revised and run for comparison (see figure 4).

```
10 FOR N = 1 TO 100 : PRINT N, 4 * N + 2 : NEXT N
20 END
```

Figure 4

We then explored what changing the "LET" statement in line 20 of figure 3 would do. For example, we tried changing the expression to the right of the "equal" sign to read

$$2 * X, 2 * X + 1, 2 * X - 1, \text{ etc.}$$

The children noticed that the numbers generated by these expressions grew much faster than the numbers generated by repeatedly adding a number to the preceding memory cell (i.e., "LET X = X + 1," LET X = X + 2," LET X = X + 3," etc.).

An interesting incident occurred during my next meeting with the children. Some visiting Egyptian educators who were interested in observing children working in our computer topics program dropped by. I showed the children and visitors the "Tower of Hanoi Puzzle" consisting of five drilled disks of increasing diameter, all stacked on one of three pegs (see figure 5). As many readers probably know, the problem is to move one disk at a time to another peg so that the disks end up in the same order as on the original stack. You are not alllowed to place a disk of a larger diameter on top of a smaller disk when moving the disks from peg to peg.

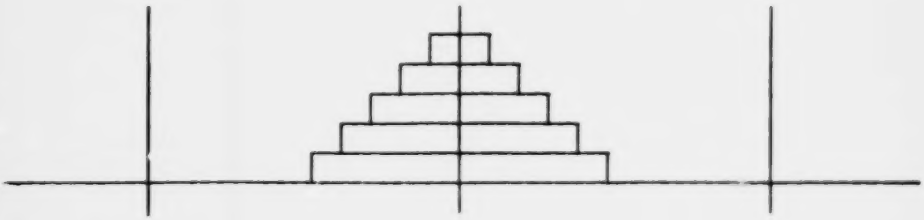

Figure 5

The children were having trouble solving the puzzle with five disks which prompted me to suggest a strategy of solving a similar but simpler problem. "Try solving the puzzle using two, three, and then four disks. Do you see a pattern? Can you extend it to five disks?"

Next I had the children record the minimum number of moves that it took to solve the puzzle using 1 through 4 disks. The table format used with the Cuisenaire Rod activity was employed again. (Figure 6)

□	1	2	3	4
△	1	3	7	15

Figure 6

I asked them if they could predict how many moves it would take for 5 disks. Several predictions were made and we verified a prediction of 31 moves by performing the moves with 5 disks. The next task posed was to extend our pattern of number pairs by predicting the number of moves required for 6, 7, 8 or any number of disks. "Does there seem to be a pattern? Can we write a computer program that will generate the number of moves for any number of disks?" Again, after guessing and checking, the following program did the trick using the "INPUT" command to enter various values for M.

```
10 LET X = 1 : INPUT R
20 LET N = 1 TO R : PRINT N, X : LET X = 2 * X + 1 : NEXT N
30 END
```

Figure 7

Now was the moment to involve the children in a problem situation in which the computer was an absolutely essential tool. I didn't feel that these children were developmentally ready to discover a formula such as $\triangle = 2^{\square} - 1$ to compute the number of moves for any number of disks in the Tower Puzzle. However, I did feel that the children could use the computer to compute any number of moves using the strictly recursive program previously listed in Figure 7. Striking while the iron was hot, I proceeded to relate the legend of an order of Buddhist monks in Hanoi whose duty was to move 64 disks from one peg of the puzzle to another following the conditions previously stated. The legend states that when the monks have completed their task the world will come to an end.

Wow! The children's eyes were popping and mouths were hanging open. "How long have we got?" one of the children asked with some concern in his voice. I proposed that we try to find out how long it would take to complete the task of moving 64 disks using a generous allowance of one second for each move of a disk to another peg.

At this point, the Egyptian visitors seemed quite taken with the interest that the children were showing in the problem and seemed curious about the outcome. They paid close attention as the children turned to the microcomputers that are stationed around the room and entered 64 for the input in the program listed in Figure 7. After a few seconds of computational time, the answer appeared on the monitor screens (1.84467441E + 19). Since many of the children had previously been introduced to scientific notation, translating the numeral on the monitor screen into scientific and standard notation provided a fine opportunity to review the use of scientific notation to represent large numbers. For the few children who were not familiar with scientific notation, it was an excellent introduction to the topic.

Of course, the children could see that $1.84467441X10^{19}$ seconds was a lot of time, but how many years does it represent? I proposed that we next write a program to convert our seconds into years and even centuries. After some discussion, the children came up with the following addition to the program in figure 7 using "M" for minutes, "H" for hours, "D" for days, "Y" for years, and "C" for centuries.

```
30 LET M = X/60 : H = M/60 : D = H/24 : Y = D/365 : C = Y/100
40 PRINT Y, C
50 END
```

Figure 8

Before the children were able to revise and run the program, the Egyptian visitors were reminded by their guide that it was time to move on for the next visit on their agenda. I couldn't help noticing that a heated discussion was taking place as one of the visitors seemed to be voicing objections to leaving. Later, the guide informed me that this particular visitor was so interested in the outcome of the conversation that he wanted to stay and see how it came out. How I wish that I could generate this degree of interest in some of my students at the University.

I won't reveal the results of the conversion. Hopefully, readers will be motivated to run to their nearest computer and try it for themselves. I will only say that the world isn't in any immediate danger from this source and the monks should be actively recruiting many generations of monks to succeed them in order to complete their task.

I hope that these activities give the reader some feeling for the points that I am trying to make in this article. An important skill for problem solvers to develop is the ability to select problem solving strategies that are appropriate for the problem being solved. A computer can be a powerful tool when solving problems using algorithmic or recursive processes. It allows children to tackle problems involving concepts which

were previously inaccessible to children in the intermediate grades prior to more formal mathematical treatments of topics such as arithmetic, geometric, and exponential sequences and series. Intermediate grade children can now explore problems involving these concepts through writing relatively simple programs using simple recursive computations thus laying an intuitive foundation for a higher level of understanding later.

Thus, using a computer as a problem solving tool allows children to explore problems requiring lengthy computations for which using paper and pencil, mental computations, or even handheld calculators would be impractical. It would have been very time consuming for the children to have calculated the number of moves for the 64 disks in the Tower Problem using our recursive approach, even with a calculator. In addition, I'm sure that the excitement of getting an answer would have been lost.

A computer can be assigned many roles in a classroom. Don't relegate it solely to the relatively passive roles of drill master, entertainer, or simulator of pre-programmed "real life" situations (mathematical or other). It has an important and active role to play in developing children's problem solving skills. Try it. You'll like it!

A BRIEF BIBLIOGRAPHY INCLUDING SOURCES OF PROBLEMS APPROPRIATE FOR COMPUTER ASSISTED SOLUTION IN THE INTERMEDIATE GRADES

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ALGORITHMIC AND HEURISTIC APPROACHES TO COMPUTER-RELATED INSTRUCTION

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In his book, *The Computer in the School: Tutor, Tool, Tutee* (Teachers College Press, New York; 1980), Robert Taylor has assembled a set of papers and articles by five leaders in the use of computers for instruction and learning. This article uses and expands on a conceptual framework (which was presented by one of the authors, Thomas Dwyer, in one of his articles) to compare and contrast the ideas, beliefs, research and development posited by Patrick Suppes, Arthur Luehrmann, Alfred Bork, and Seymour Papert in their articles on computer-related instruction.

Thomas Dwyer suggests that there are two systematic approaches — algorithmic and heuristic — for translating educational theory into large-scale practice. The algorithmic approach is "oriented toward defining predetermined sequences of activities for achieving predetermined measurable goals." (p. 89) This approach — an outgrowth of the behaviorist movement and the field of systems analysis — gives structure and precision to the instructional process and increases the probability of reproducibility of results by emphasizing convergence. Most decisions are made by someone other than the learner and the teachers are usually viewed an instructional managers in this "bottom up" approach to instructional design.

On the other hand, heuristic approaches are more open-ended in that "neither output (objectives) nor algorithm (processes) are completely specified." (p. 90) Heuristic strategies are designed "top down" to help learners (1) be aware of and move toward global goals, (2) make decisions and discoveries, and (3) learn how to learn what to do (rather than just to learn what to do). These strategies, which emphasize divergent thinking and behavior on the part of the learner, require that the teacher be a guide, a co-learner, and a resource person to be managed by the other learners.

Algorithmic and heuristic approaches can be assigned to different ends of a continuum against which selected authors who have views on curriculum and instruction (in this case, computer-related instruction) can be plotted to illustrate their relative positions. Figure 1 shows how Patrick Suppes, Arthur Leuhrmann, Alfred Bork and Seymour Papert compare — in a gross sense — on the "algorithmic/heuristic" continuum. This Figure also illustrates graphically how four major curricular and instructional factors vary according to their location on the continuum.

A purely algorithmic approach would be relatively restrictive in regard to (1) the array of curricular goals available to the learner, (2) the amount and type of individualized instruction that would occur, and (3) the involvement of the learner in making decisions about curriculum and methodology. The teacher would play the major role in the decision-making processes about curriculum and instruction and would most likely be highly directive. On the other hand, at the heuristic end of the continuum, the (1) curriculum goals would be quite diverse, (2) individualization of instruction would accommodate a variety of learner differences such as rate and style of learning, preferred methods and individual goals, and (3) learners would participate actively in the decision-making. The teacher would help learners make decisions rather than make decisions for them.

The remainder of this article discusses and illustrates — with an emphasis on curriculum goals, individualization of instruction, and decision-making — the relative role of algorithmic and heuristic approaches as described by Suppes, Luehrmann, Bork, and Papert in their selected papers/articles.

Patrick Suppes

The algorithmic approach plays a dominant role (as compared to heuristic approaches) in Suppes' concept of individualized drill-and-practice. Suppes feels that computer-assisted instruction can be used as a supplement to the "regular teaching process. After the teacher has introduced new concepts and ideas in the standard fashion, the computer provides regular review and practice of basic concepts and skills." (p. 232) He goes on to contend that computer-assisted drill-and-practice is quite appropriate for helping learners succeed in the skill subjects which, he notes, *presently comprise a major part of the school curriculum.*

The curriculum, viewed in the drill-and-practice framework, is largely determined by someone other than the learner. It could be the teacher, the textbook author, or a curriculum committee — with little or no input from the learner. The individualization of instruction that would occur would primarily account for learner differences in rate of learning and achievement level. Basically, all learners would study the same curriculum in the same way, just at different rates and from different entry points (but with the same exit points). The teacher's role would be modified only slightly from the present because he/she would act primarily as an "instructional manager."

Arthur Luehrmann

Luehrmann's position on the "algorithmic/heuristic" continuum portrayed in Figure 1 is to the right of Suppes. While Luehrmann asserts that computing "constitutes a new and fundamental intellectual resource" (p.

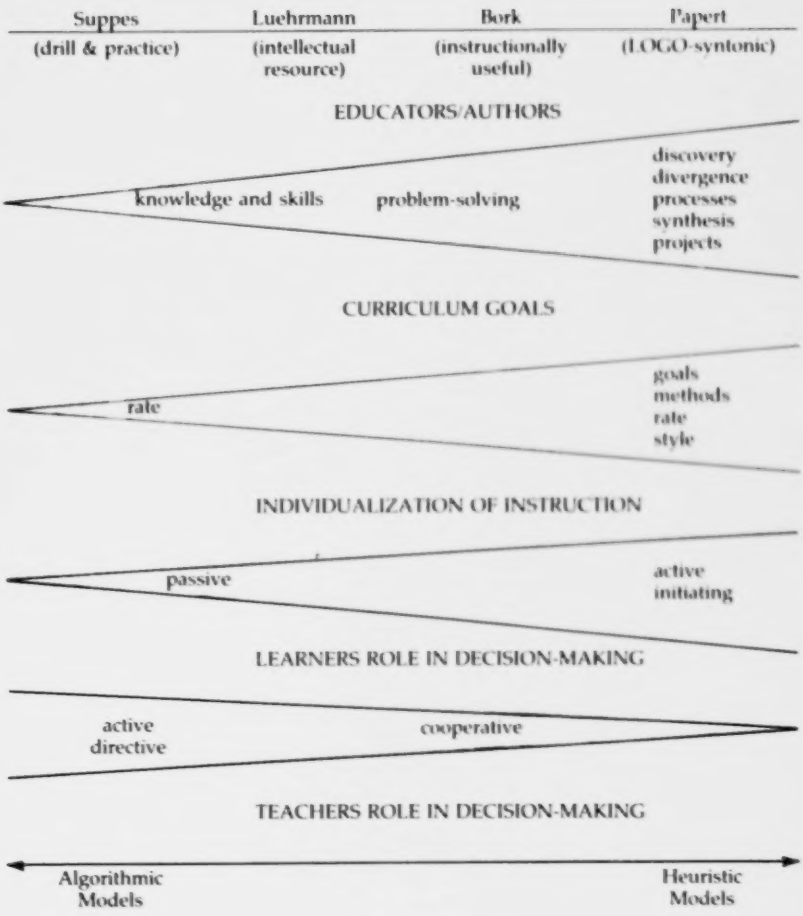

Figure 1
Relative Positions of Four Educators/Authors (Suppes, Luehrmann, Bork, Papert) and a representation of the Relative Amounts/Types of Curriculum Goals, Individualization of Instruction, Learners Role in Decision-making, and Teachers Role in Decision-making in reference to an Algorithmic/Heuristic Continuum

133) and feels that to use computers to simply deliver instruction would be a drastic mistake, he does feel that computers and associated technology can be used in an algorithmic fashion to help teach basic knowledge and skills. Computer-assisted instruction (CAI) and computer-managed instruction (CMI) are highly compatible with present-day school systems and can be used for drill, practice, and remediation items

which require much teacher time but are relatively unattractive and unrewarding for most teachers. He sees CAI facilitating the individualization of instruction by helping adapt instructional processes and resources to learner differences in terms of learning style, rate of learning, and preferred alternate routes toward mastery. In addition, CMI can facilitate teacher planning, organization and record management.

Luehrmann sees the need for heuristic approaches when he asks if "the computer is so powerful a resource that it can be programmed to simulate the instructional process, shouldn't we be teaching our students mastery of this powerful intellectual tool?" (p. 135) He asserts that computers and computer technology should be used both as the "means" and the "end" of new courses which would be designed to teach students high-level problem solving skills (including data acquisition, analysis, manipulation, interpretation, and presentation), decision-making, and simulation design and development. Within the framework of such courses, there would be opportunities for much hands-on experiential work on the part of the students to supplement standard book-oriented, classroom discussions.

The curriculum, as Luehrmann sees it, would be determined largely by someone other than the learner, but would be comprised of higher-order intellectual skills in addition to the lower-order skills addressed by Suppes in his drill-and-practice-oriented efforts. Teachers would still be instructional managers for a major part of their time, but would also spend time and effort in the instructional design and development of new courses that would need to be implemented to help learners acquire the higher-order problem-solving skills. The amount and type of individualization of instruction that would occur would accommodate differences in style and alternate routes in addition to "rate" and "entry point." All learners would study the same curriculum but in possibly different ways and at different rates starting at different entry points (but all exiting at the same place).

Alfred Bork

Bork's position on the "algorithmic/heuristic" continuum (see Figure 1) is to the right of Luehrmann and illustrates that he puts more emphasis on heuristics than on algorithmics. He emphasizes that experiences (play, for example) are unique for each individual and that each of us "does different things in the environment and obtains different results, even in group situations." (p. 55) Therefore, he argues that considerable emphasis should be placed on the design of "instructionally useful experiences" that will help learners acquire problem solving skills. He emphasizes that learners need to do more than just see the "given" elements of a problem, some finely polished process for solving the problem, and then the final solution. He feels that the learner needs to struggle with

"false starts" and needs to analyze his/her own problem solving processes in order to refine them to become better problem solvers. He contends that the heuristic strategy of using computer-based dialogues, while currently experimental and untested, provides a potentially highly useful avenue for helping students acquire such skills.

However, in all of his emphasis on the use of heuristic strategies, he does not lose sight of the place of algorithmic strategies when they are needed. For example, he contends that on-line tests have much potential in that they enable learners to make responses, receive feedback on those responses, and then receive remedial aid as needed. Bork feels that algorithmic approaches — such as textbooks, lectures and other one-way media which tend not to help students become actively involved in problem solving — are necessary, but not sufficient, for reaching the educational goals he values.

The curriculum, in Bork's conceptualization, is still largely determined by someone other than the learner. It would include provision for learners to acquire higher-order problem-solving skills with the opportunity for them to deviate from the prescribed curriculum at times. Within Bork's framework, teachers would spend more time on instructional design and development activities than they would in Suppes' or Luehrmann's frameworks. They would design, in cooperation with teammembers (other teachers, programmers, and typists), simulations of reality that would stretch the intellect and imagination of the learners. The amount and type of individualization is considerably more than in Suppes' and Luehrmann's conceptualization. Through the considerable use of graphics, Bork illustrates a mechanism that recognizes that students do learn in different ways (p. 34) and approaches which are successful for one student might not be beneficial for others. Naturally, his notions for individualization include accommodations for learners' differences in rate of learning, entry point, and need for remediation.

Seymour Papert

Papert is the most radical of the four authors in that his concept of LOGO-based syntonic learning is on the opposite end of the "algorithmic/heuristic" continuum from Suppes. Papert asserts that "the computer has brought us the technological infrastructure that can make possible a real intervention in the learning environment, one in which syntonic learning (a heuristic) can be generalized to cover a much greater area (than just mathematics)." (p. 202) He makes a strong case for syntonic learning — learning in which the learner establishes a strong relationship (or identity) with the content — as opposed to dissociate learning in which the content (in his examples, mathematics and physics) has no relationship to what the learners "enjoy, or love, or do, or appreciate." (p. 202)

Papert contends that present instructional practices (which are mostly algorithmic and dissociate in nature) inhibit the possibility of syntonic learning for most students by (1) failing to help them relate new content to already acquired or experienced content, (2) contradicting much of their own experience of how things in their world really work and by failing to allow or provide a synthetic overview, and (3) utilizing a passive mode of learning in which they are not allowed or enabled to assimilate the new knowledge in a personally meaningful way. To provide a way to overcome these weaknesses, Papert suggests we utilize a heuristic learning theory model consisting of microworlds "in which active learning consists of exploration by the learner of a microworld sufficiently bounded and transparent for constructive exploration and yet sufficiently rich for significant discovery." (p. 208)

The curriculum, according to Papert, would consist largely of computer-enhanced microworlds which would facilitate syntonic learning by enabling learners to actively explore microworld(s) in which "hooks" exist to relate preformal, intuitive knowledge to formal, rigorous knowledge. The curriculum would emphasize a project-oriented approach instead of the problem-oriented approach in order to capture the learners' time on the task(s) — "time to try different approaches to sub-problems, . . . to talk about it, . . . to develop a common language with a collaborator or an instructor, to relate it to other interests and problems." (p. 179) The project orientation would also enable learners to see the big picture, "including critical phases such as planning, trying simple cases, finding simple solutions, debugging them and using an iterative process to complete the project." (p. 180)

The teachers who use the microworld model for instruction will behave in quite different ways from those teachers who follow a planned, algorithmic-type curriculum. As Papert sees it, the "microworld teacher" would introduce students "to the microworld in which discoveries will be made, rather than to the discovery itself." (p. 209) The individualization that occurs within Paperts' LOGO-based syntonic learning framework is the most comprehensive individualization plan presented by the different authors. Learners have the ability to explore and develop their own routes to their own goals (within the bounds of the microworld) at their own rates in ways that are personally meaningful to them.

Implications

The viewpoints presented by these leaders in computer-related instruction and learning are neither "right" nor "wrong." Rather, they represent a range of positions that can be useful to mathematics teachers, curriculum committees and supervisors as they struggle to determine "where they are" and "where they want to be" in reference to the mathematics curriculum and the use of computers for instruction. If the mathematics

curriculum and associated instruction is largely oriented toward the acquisition of skills and knowledge by students, as many researchers suggest, should we simply use the computer to enhance the status quo? Or, since the computer has the capability to help students work at higher intellectual levels, shouldn't we seize this opportunity to use it to help us help our students achieve long-sought but rarely achieved goals such as divergent thinking, creativity, insight, discovery, and initiative? The decision is yours. The things you do in your schools and classrooms will set much of the framework for your students' future behavior in and attitudes toward mathematics.

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CALCULATORS: ARE THEY REALLY USED IN THE CLASSROOM FOR PROBLEM SOLVING?

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Ever since hand held calculators made their appearance in the market several years ago, individuals began to use them for everyday arithmetic problems such as balancing a check book, adding up the grocery list, finding the mileage of their automobiles and occasionally calculating the area of a room in square feet to lay a carpet. Obviously, most of these calculations require nothing more than addition, subtraction, multiplication, and division and rarely, the finding of a percent of a number or a square root. Pocket calculators can be found not necessarily in someone's pocket but in briefcases, ladies' purses, on peoples' desks or the kitchen counter.

Math teachers, on the other hand, hesitatingly and timidly began to use the calculators for checking answers after the students had completed the painful task of 4-digit-by-4-digit multiplication or long division problems. Even after the publishing of the NCTM "An Agenda for Action" where item #3 specfically stated that calculators were to be used extensively, most teachers still feel that it is a taboo, a sort of cheating if their students use the calculator in class or to complete their homework. Even mathematics teachers teaching trigonometry insist that students first must know how to interpolate before they can use the calculator to find the trig functions or logarithms.

One of the reasons calculators are not used in the classroom and students are often discouraged from employing this powerful tool is because teachers did not have any training in the use of calculators nor did they spend the time to discover and understand the tremendous algorithmic power of those little machines.

By mere observation one may wonder, for example, why there are 40 keys on the TI 30 scientific calculator if only about 15 are used by most people. In addition, those 40 keys perform 51 different functions, yet most of us use 4 or 6 functions. Calculators have a great potential in the mathematics classroom if their powers are properly utilized.

To demonstrate the powers of the pocket calculator and its practical uses in the classroom, I list 4 simple examples with their solutions. Notice that no interruption is necessary to write down an answer before the user can perform the next operation. Additionally, the potential of real problem solving capabilities is incorporated in these examples which include planning, application, and evaluation (estimation). Analysis and synthesis, on the other hand, take place in the planning stage.

CAUTION: Do not attempt to remember steps. Such a task will be more cumbersome than long division. Simply instruct students to follow the principles of mathematics such as working first with the quantities inside the parenthesis or under the radical, etc. Furthermore, there is no single solution to a problem using the calculator. That alone is a significant opportunity for students to explore and investigate the basic ingredients in problem solving.

ACTIVITY I

DOWNHILL SKIING

In the 1984 Winter Olympics in Sarajevo, Yugoslavia, Bill Johnson from Southern California was the first American to win a gold medal in the downhill Alpine skiing. His recorded time was 1 minute, 45.59 seconds. The downhill course measured 1.9 miles. What was Johnson's average speed in miles per hour?

PLANNING:

1. Formula to be used: $d = t \cdot r$, where
   d = distance in miles
   t = time in hours
   r = rate, mph
2. Solve for $r = \frac{d}{t}$
3. In this problem: $d = 1.9$ miles
   $t = 1$ minute 45.59 seconds
4. Substituting the values in the formula we have:
   $$r = \frac{1.9}{1' \ 45.59''}$$
5. First express 1' 45.59" in hours.

APPLICATION: (Use TI 30 or TI 30 II)	PRESS	DISPLAY
	60; +; 45.59; =	105.59
	÷; 3600; =	0.0293306
	STO; ON/C	0
	1.9; ÷; RCL: =	64.778862

Johnson's average speed was 64.78 mph.

EVALUATION: (Estimation)

Is 64 mph in downhill skiing a reasonable answer?
Discussion.

ACTIVITY II

HERON'S FORMULA

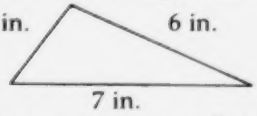
Heron was a Greek mathematician who lived in Alexandria in the 1st century A.D. He developed a formula to determine the area of triangle from the measures of it's sides.

$A_{\triangle} = \sqrt{s(s-a)(s-b)(s-c)}$, where a, b, c are the lengths of the sides and

$s = \frac{a+b+c}{2}$, i.e. the semiperimeter of the $\triangle$.

The formula, for many years, was not in use because of its complicated and cumbersome calculations. Using the calculator, however, the computations can be done in a matter of seconds.

EXAMPLE: Find the area of a triangle whose sides are indicated in the figure.



PLANNING: 1. Find s(semiperimeter): $\frac{3 + 6 + 7}{2} = 8$

2. Substitute the given values in the formula.

$$A_{\triangle} = \sqrt{8(8-3)(8-6)(8-7)}$$

3. One must work first with the quantity under the radical in the order given, then take the square root of the product.

APPLICATION: (Use TI 30 or TI 30 II)	PRESS	DISPLAY
	8; x; (; 8; -; 3;); x; (; 8; -; 6;); x; (; 8; -; 7;); =; $\sqrt{x}$	8.9442719

The area of the triangle rounded to the nearest hundreth is: 8.94 sq. in.

EVALUATION: (Estimation)

Is the answer in the "ball park?" For a rough estimate we may use 6 as the base and 3 as the altitude. Then, substituting in the formula for the area of the triangle, we have:

$A_{\triangle} \approx \frac{1}{2}(6 \times 3) = \frac{18}{2} = 9$

The answer is, indeed, in the ball park.

ACTIVITY III

SUMMER OLYMPICS, L.A., 1984

Carl Lewis broke the Olympic record in the 200 meter dash in 19.80 seconds previously held by Tommie Smith (19.83 seconds). What was Lewis' average speed in miles per hour?

PLANNING: 1. Formula to be used: $d = t \cdot r$, where
d = distance in miles or kilometers
t = time in hours
r = rate, mph or kmph

2. Solve for $r = \frac{d}{t}$

3. Since the distance is given in meters, we may use kilometers for d, then convert the result (rate) into miles.

4. Substituting in the formula, we have:
$r = \frac{200\ m}{19.80''}$

5. First express meters into kilometers which is $\frac{200}{1000} = .2$ km
Also, express 19.80 seconds into hours.

APPLICATION: (Use TI 30 or TI 30 II)	PRESS	DISPLAY
	19.80; ÷; 3600; =	0.0055
	STO; ON/C	0
	.2; ÷; RCL; =	36.363636

EVALUATION:
(Estimation) Lewis' average speed was about 36km per hour. To convert to miles divide by 1.609 (1 m = 1.609 km)
Answer: 22.600147 mph. Is this a reasonable answer? Can a man run 22 mph?
Discussion.

ACTIVITY IV

SAILING

The Olympic rate of a sailing yacht is a function of several variables.

$$R = 0.9\left(\frac{L\sqrt{A}}{12\sqrt[3]{V}} + \frac{L + \sqrt{A}}{4}\right) \text{ where}$$

L = length of yacht in meters
A = surface area of sails in square meters
V = volume of water yacht displaces in cubic meters

To be in the R-5.5 class at the Olympics, a yacht must have an Olympic rating less than 5.5. Does a yacht that is 10.5m long, has $38m^2$ of sail, and displaces $10m^3$ of water qualify?

PLANNING: 1. Substitute the given values in the formula.

$$R = .09\left(\frac{10.5\sqrt{38}}{12\sqrt[3]{10}} + \frac{10.5 + \sqrt{38}}{4}\right)$$

1. Knowing the functions and memory in the calculator one must begin with $\sqrt[3]{10}$ or $\sqrt{38}$.

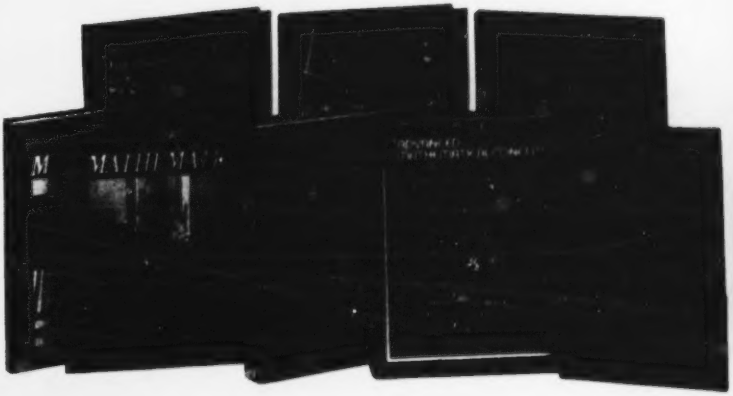
APPLICATION: (Use calculator TI 30 or TI 30 II)	PRESS	DISPLAY
	10; INV; Y^x; 3; =; x; 12; =	25.853216
	STO; ON/C	0
	38; $\sqrt{x}$;	6.164414
	x; 10.5; =	64.726347
	÷; RCL; =	2.5036091
	STO; ON/C	0
	38; $\sqrt{x}$;	6.164414
	+; 10.5; =; ÷; 4; =	4.1661035
	+; RCL; =	6.6697126
	x; 0.9	6.0027413
	Yacht does not qualify.	

EVALUATION: (Estimation)

Since the answer is in the "ball park" the computations were correct. To double check, repeat the process.

Discussion: In order to reduce the number to less than 5.5 what variables(s) must be increased or decreased?

You know what you success . . . so does



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PROBLEM SOLVING AND THE NUMBER OPERATIONS

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The National Council of Teachers of Mathematics recommends that problem solving be the focus of school mathematics in the 1980s. The supportive arguments for the inclusion of problem solving in the mathematics curriculum have been well-founded. Yet, any impact that problem solving will have on mathematics education will depend on the teacher. He/she is the catalyst in the students' development of problem-solving skills. It is not only necessary for the effective teacher to provide instruction of appropriate problem-solving strategies, but also to give students ample opportunity to apply these strategies.

Number operations provide much opportunity for problem-solving activities. This involves the use of addition, subtraction, multiplication, and division in unique ways. The following examples allow students to (a) practice their number operations skills and (b) use insightful reasoning and problem solving.

I. Magic Squares

An additive magic square (e.g., 3 x 3) is a matrix consisting of nine real numbers in which the sum of any three numbers — horizontally, vertically, and diagonally is the same. In the following magic square, each sum is 15.

8	1	6
3	5	7
4	9	2

Now add 5 to each digit. Do you get another magic square? YES!!

13	6	11
8	10	12
9	14	7

Multiply each digit by 2. Do you get another magic square? YES!

16	2	12
6	10	14
8	18	4

If you want practice with negative numbers, then subtract 4 from each digit.

4	-3	2
-1	1	3
0	5	-2

If you want work with fractions, divide each digit by 3.

8/3	1/3	2
1	5/3	7/3
4/3	3	2/3

So, if you add, subtract, multiply, or divide each digit by the same number, you will again get a magic square. As a matter of fact, if you add two magic squares together (by position), you will have another magic square.

Ask your class to generate magic squares of different sizes (e.g., 4 x 4) and different sums.

II. Making Digits

Using *four* 4's and the number operations, construct the numbers 1-10.

Example: $1 = \frac{44}{44}$

$2 = \frac{4}{4} + \frac{4}{4}$

$3 = \frac{4+4+4}{4}$

$4 = 4 - \frac{(4-4)}{(4)}$

Ask your class to use other number combinations.

III. Cryptarithmetic

Consider the following problem:

$$\begin{array}{r} AA \\ +BB \\ \hline CBC \end{array}$$

Here each letter represents a digit (0, 1,...9). The first digit in a number will never be zero. Hence $A \neq 0$, $B \neq 0$, $C \neq 0$. Such puzzles are called cyrptarithmetics, all involving the assignment of distinct digits to distinct letters, to represent some arithmetic problem.

Students must use insightful reasoning as well as their knowledge of the number operations to these puzzles.

In the example above, we know $C = 1$, since if you add two 2-digit number, the hundreds place will be either 0 or 1, and since $C \neq 0$, then $C = 1$.

Now we have $\begin{array}{r} AA \\ BB \\ \hline 1B1 \end{array}$

Now $A + B$ must add to 11. Therefore, the combinations left are ($A = 6$, $B = 5$) or ($A = 7$, $B = 4$), or ($A = 8$, $B = 3$), or ($A = 9$, $B = 2$). It could be vice-versa. The answer is $A = 9$ and $B = 2$, or $\begin{array}{r} 99 \\ +22. \\ \hline 121 \end{array}$

So finding the answer is *not* simply trial and error. Ask your class to find values for the following and also ask them to explain their reasoning.

$$\begin{array}{r} HALF \\ +HALF \\ \hline WHOLE \end{array} \qquad \begin{array}{r} MOM \\ +POP \\ \hline LOVE \end{array} \qquad \begin{array}{r} WRONG \\ +WRONG \\ \hline RIGHT \end{array} \qquad \begin{array}{r} HOCUS \\ +POCUS \\ \hline PRESTO \end{array}$$

The class can also practice other number operations. These are more difficult, however.

$$\begin{array}{r} EIGHT \\ -FIVE \\ \hline FOUR \end{array} \qquad \begin{array}{r} ABCD \\ \times 9 \\ \hline DCBA \end{array}$$

$$\begin{array}{r|l} & HIL \\ \hline IL & PHIL \\ & IL \\ \hline & TI \\ & LS \\ \hline & HIL \\ & HIL \end{array}$$

IV. DIGIT DETECTIVE

Mathematics teachers at all levels are always looking for new and interesting ways to have students practice necessary computational skills. One such activity that provides this practice is digit detectives. This activity can be used at different grade levels and encourages students to use problem solving skills. Here are some sample problems:

Directions: Replace each blank with a digit so that the solution to the exercise will be correct.

1)
$$\begin{array}{r} _28 \\ +2__ \\ \hline 987 \end{array}$$

2)
$$\begin{array}{r} 41_ \\ -2_4 \\ \hline -_85 \end{array}$$

3)
$$\begin{array}{r} _6 \\ \times 2_ \\ \hline 92 \\ _2 \\ \hline _1_ \end{array}$$

4)
$$\begin{array}{r} 8_2 \\ 7\overline{)56_4} \\ \underline{\quad\;} \\ \quad\;\; \\ \underline{\quad\;} \\ 0 \end{array}$$

This activity does not have to be limited to whole numbers. Two examples below use fractions:

5) $\frac{3}{5} \times __ = 27$

6) $\frac{_}{3} \times \frac{_}{_} = \frac{4}{5}$

You may want to encourage your students to use calculators for the more difficult problems.

7)
$$\begin{array}{r} __ \\ \times 5\,4 \\ \hline ___ \\ \hline 3_34 \end{array}$$

8)
$$\begin{array}{r} \quad\;\; _3_ \\ __\overline{)3_\;_8} \\ \quad\;\; \underline{=\;=} \\ \quad\;\; 8_ \\ \quad\;\; \underline{_2} \\ \quad\;\; 1_8 \\ \quad\;\; \underline{1_8} \\ \quad\;\; 0 \end{array}$$

V. WHAT IS MY RULE?

For each of the following, fill in the blanks and state the rule:

		Rule
1)	2,4,6,8,_,_	Add 2 to preceeding number
2)	3,6,9,12,___,___	Add 3 to preceeding number
3)	3,15,75,375,____,____	Multiply each number by 5 to find next number.
4)	2,3,5,9,17,___,___	Multiply by 2 and then subtract 1 to find next number.
5)	4,6,10,18,___,___	Subtact 1 and then multiply by 2 to find next number.

Have each student make his/her own rule and challenge the rest of the class to find the rule. Here are some more difficult problems.

6) 3,9,81,65,61,___,___
Hint: $3 \times 3 = 9$, $9 \times 9 = 81$, $(8 \times 8) + (1 \times 1) = 65$, . . .

7) 1,7,19,43,___,___
Hint: $[(19 + 4) \times 2] - 3 = 43$

VI. NUMBER SEARCH

I would now like to present a series of problems (known as number search problems) which clearly and succinctly illustrate a problem-solving model. More importantly, these problems and strategies for solving them may be presented and discussed in mathematics classes in both elementary and secondary schools.

Consider the following functions:

F: $x - 7$

G: $x + 11$

H: $2x + 1$ if x is even
$\frac{x - 1}{2}$ if x is odd

J: $\frac{x}{3}$

K: $2x$

M: $2x + 1$

N: $\frac{x + 1}{3}$

Thus, the effect of applying F to a number (x) is to subtract seven; the effect of applying G to a number is to add eleven; the effect of appying K to a number is to double the number and so on.

Problem: Given the number 99 (initial state), try to generate the number 100 (goal state) using the operators F and G.

(Hence, a number search problem involves the construction of a solution path from a starting number to an ending number using successive applications of given functions.)

Solution: 99 G 110 F 103 F 96 G 107 F 100

This is but one possible solution path. There are many others. We are certain your students will generate many solution paths.

Many elementary school students in solving this problem verbalized their strategy as: when the number was too small (less than goal), add (apply G), and when the number was too large (more than goal), subtract (apply F). This strategy makes perfect sense and, will in fact, produce the most efficient solution path. Secondary school students sometimes represent the problem as $11x - 7y = 1$ and apply the Euclidean Algorithm. Others try to find two equations in two unknowns (not possible unless one is told the minimum number of steps). Hence, this problem is a great equalizer.

Other number search problems include:

Initial	*Goal*	*Operators*	*Minimum Number of Steps*
1. 36	30	F,G	6
2. 90	80	J,K	5
3. 500	300	M,N	7
4. 107	17	G,J	7
5. 6	148	G,H	6
6. 100	17	G,H	6
7. 1776	50	G,H,J	5

Mathematics teachers can use these and similar problems to discuss many specific and general problem-solving strategies (heuristics). For example, in problem 4 above, a solution path beginning with 107 J $\frac{107}{3}$ . . . will *never* lead to the goal of 17. One can never eliminate the decimal ($\frac{107}{3} = 35.66$) by applying G or J successively. Hence, a specific strategy is never to apply J unless the number is evenly divisible by three. A general strategy could involve working backwards (e.g., starting with the number 17, generate the number 107 using the inverses of G and J). Many students appear to work forwards and backwards by setting subgoals, another strategy.

It is imperative that we discuss these and other strategies with our students. Strategies assist people in constructing correct and efficient

(minimum) solution paths and thus lends credence to their power. Hence number search problems can pay large dividends, and yet, require very little content background.

PROCLAMATION

National Council of Teachers of Mathematics

WHEREAS, *mathematical literacy is essential for citizens to function effectively in society; and,*
WHEREAS, *mathematics is used every day - in the home, on the job, and for recreational activities; and,*
WHEREAS, *the language and processes of mathematics are basic to other areas of the school curriculum; and,*
WHEREAS, *expanding technology demands increased mathematical competence; and,*
WHEREAS, *relevant community and school activities can generate interest in mathematics;*
NOW, THEREFORE, *I, F. Joe Crosswhite, president of the National Council of Teachers of Mathematics, do hereby proclaim the week of 14-20 April 1985 as*

MATHEMATICS EDUCATION WEEK

to be observed in schools and communities in recognition of the importance of mathematics.
IN WITNESS WHEREOF, *I have hereunto set my hand and caused the Corporate Seal of the National Council of Teachers of Mathematics to be affixed on this 1st day of November 1984.*

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MATH ANXIETY: WHAT ARE WE DOING TO OUR STUDENTS?

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As we enter the typical secondary school, things seem calm and quite ordinary. A group of students has gathered around a bulletin board where newspaper clippings from the homecoming game are on display, couples are strolling down the corridor, several boys are standing against the wall watching the girls go by while two of their objects of interest are giggling as they hang rock stars' pictures in their lockers. A few teachers are standing by their doors while another, a little late, is hurrying toward the faculty room. Everyone is relatively happy and appears confident about the upcoming day.

Now it is time for first period and our focus is on the mathematics classrooms. Walking down the hall and glancing through the windows, we notice that most of the students are attentive. However, here and there, a boy has his head down, another is staring out the window, a girl is writing a note, someone else is headed for the pencil sharpener, and another is trying to get a neighbor's attention. A door opens and student exits with a pass to the nurse's office in hand. Are these people uninterested in mathematics? Are they trying to irritate the teacher by interrupting class or by causing trouble?

Such explanations might account for student behavior occasionally in any situation. When observed repeatedly in the same subject area setting, however, they may well be symptoms of anxiety. What is math anxiety and is there any need for us to be concerned about it?

Bonnie Donady and Sheila Tobias (1977) define math anxiety as a "nonrational distaste for and avoidance of math and math related subjects" (p. 71). Dr. Wade Sherard (1981) of Furman University describes it "as a fear of mathematics or an intense, negative emotional reaction to mathematics" (p. 106).

Based on these definitions, it does not seem that math anxiety is a new problem. People for many years have expressed a dislike for and an avoidance of mathematics. Their lives have gone on and they have functioned in and contributed to society just the same. The question is, could these people have had more choices for careers and in turn more control over their own lives if they had not avoided mathematics.

Lucy Sells (1978), who has studied the link between mathematics and vocational opportunities, calls math the "critical filter" which tends to

limit people from pursuing many occupations. Mastery of algebra alone enables people to do better on civil service exams or industrial entrance exams resulting in better starting jobs, higher wages, and the potential for on-the-job training. Another consideration is that often guidance materials are misleading since they list careers which explicitly require math at the outset only. Students choose one of these fields believing they will be able to advance to higher positions only to learn midway through their careers that this is impossible without a knowledge of higher level mathematics. Thus, ignoring students who have math anxiety to the extent that they will avoid taking math beyond basic graduation requirements allows them to limit severely the choices they will have for careers and for college majors.

In 1973, Sells found that only 43% of the males and 8% of the females enrolled in the freshman class at Berkeley had taken four years of high school math — resulting in 57% of the men and 92% of the women being excluded from 75% of the majors (Donady, 1977). Currently, of the ten colleges at The Pennsylvania State University, five require three years of high school mathematics with the remaining ones requiring four years. Then, once enrolled, the students must take at least six credits of math to meet graduation requirements — not a very pleasant thought for anyone with math anxiety. Students who avoid taking more than just general mathematics in high school are eliminating themselves from acceptance into many specific colleges and majors, perhaps before they have even begun to think seriously about future plans.

In addition to limiting career options, people suffering from math anxiety may have difficulty in dealing with everyday math-related events. Using a checkbook, making change, or calculating a tip are all beyond the range of what a math-anxious person can do comfortably, if at all. Income tax reporting time can be traumatic. Just receiving a W-2 form and having to gather records to take to a tax-preparer can produce a negative reaction.

If we care about *all* of our students and their occupational futures, as well as their ability to cope with everyday living, then we must be concerned about math anxiety.

Those of us who do not think anxiety is much of a problem should consider the fact that universities throughout the nation have seen a need to establish math clinics. Secondary teachers can gain important insight into this problem by examining what has been learned from students who have attended math clinics. One technique that Donady and Tobias (1977) as well as Stanley Kogelman and Joseph Warren (1978) use is to have the math-anxious person keep a diary of how he feels when he is doing math as well as when he is avoiding math. Through diaries, autobiographies, and interviews, they have found that students remember being taught in an atmosphere of tension with an emphasis on right answers and the

demands of timed tests. They observed their teachers become impatient and frustrated by the challenge of trying to get students to understand "simple" ideas. They can recall being isolated at the board, having a fear of being "too dumb or too smart," being told they did a problem the wrong way, and having to justify every single thing in math class — intuition had no place.

To discover whether or not these memories were figments of peoples' imagination or perhaps just isolated cases, the author conducted a survey of 326 secondary students in a junior-senior high school during the fall of 1984. These students, 178 males and 148 females, ranged from seventh graders to seniors and from all tracks of general math to algebra I and algebra II. They were asked to relate any unpleasant experiences that they had in math class (anonimity was guaranteed). Since the question was open-ended and answered by many who were comfortable studying mathematics, responses were quite varied. Several never had any unpleasant experience, while others related embarrassing incidents including tripping, forgetting an assignment, getting detention, having to sit in the back of the room for talking out of turn, having the teacher take a note intended for a girlfriend, and being vomited on in fifth grade. The most devastating experience may have been that of a seventh grade boy who had the length of his hand compared to that of a girl's.

However, there were more serious responses. One girl remembered having a great amount of difficulty in fifth grade when she was not in a special math class. Now, in a regular seventh grade class, she is afraid her trouble with math will recur. An eighth grader recalled being caught daydreaming and having an eraser thrown at her. A ninth grade algebra student had the unpleasant experience of being laughed at during the two previous years whenever she didn't understand. Another still thought about how he hardly ever understood mathematics in sixth grade and was invariably called on when he didn't. A sophomore related that she never understood math since sixth grade and her teachers never cared. Other responses included: "When I didn't understand the easiest problem on the page"; "I did my homework, but still failed the tests"; "When I didn't know the difference between the less than and greater than problems in 5th grade"; ". . . made fun of when asked or answered a question. . . . couldn't do multiplication facts when I used to know"; "When I had to go to the board to do a problem I didn't have done. . . . It took me 15 minutes to do."; "everyday is an unpleasant experience." This second set of comments came for the most part from students who seemed to have tendencies toward math anxiety as determined by their consistent negative responses on the math anxiety self-test (Scholastic Math, 1984) and to the statement "Whenever it is time to do math, I feel _____." Fifty-two, or approximately 16% of those surveyed, fell into this category.

Whether or not these experiences, and others like them, will have an effect on the learners' achievement or continuance in mathematics is difficult to say. What is "only" unpleasant to one person may be traumatic to another. The fact is, although our sample was small compared to the number in that age group in the entire United States, these students recalled many things which to them were unpleasant, which they related to math, and which are similar to the experiences of math-anxious people now seeking help in college math clinics. It could very well be the case that insensitive teacher and class behaviors have serious and long-lasting effects on some students. It is alarming to find that 43% of the sample do not intend to take math "ever again" once they have met basic requirements for graduation. Sixty-three percent of the math-anxious plan to avoid taking more math. Thus a substantial number of young people, both anxious and non-anxious, will be limiting severely their occupational potential.

What can we as math educators do to combat this problem of math anxiety with its serious consequences? Perhaps the primary obligation of each teacher is to provide an atmosphere which is as free from anxiety-producing behaviors as possible. Constant awareness and common sense are important in establishing a positive classroom environment. The four sources of anxiety, time pressure, humiliation, an emphasis on one right answer, and isolation, delineated by Tobias (1981), are useful guides for the math teacher whose goal is to create a positive atmosphere. Time pressure on tests or in competitions can cause negative impressions for slower students. Replacing timed exercises by problems which encourage students to think and to be persistent instead of racing the clock may remove anxiety-producing situations for some. Since no one at any age likes to be humiliated, any situation which may cause it should be avoided. Adolescents are especially sensitive and need to have positive experiences to demonstrate their strengths, not their weaknesses. To leave a student struggling at the board is unkind and may be a devastating, unforgettable experience. We need to build confidence, not break it down.

Overemphasizing one right answer can not only contribute to destroying confidence but it also ignores the all-important thought process involved. Telling students simply that their answers were wrong without giving any attention to the problem-solving process discourages the use of intuition and gives rise to fear of making a mistake. Accuracy in math is important, certainly, but not to the extent that imagination, creativity, and process are stifled.

According to Tobias, the fourth area of anxiety, working in isolation, can cause stress. If learners do not know how to do a problem, sitting alone quietly will not help them learn. Working together, helping each other, and sharing ideas can develop confidence and a sense of cooper-

ation, as well as remove the stress of trying to struggle alone in solving problems.

Morris (1981) and Sherard (1981) also stress creating a positive, supportive classroom environment and helping students develop confidence in their mathematics ability. The importance of the thought process involved in arriving at a conclusion and of the use of intuition should be emphasized. They, too, think insensitive teaching behaviors which cause humiliating experiences should be avoided.

These ideas are consistent with the NCTM's *Agenda for Action* (1980) which places problem-solving as the focus for the 1980s. "Fundamental to the development of problem solving ability is an open mind, . . ., the willingness to probe, to try, to make intelligent guesses" (p. 3). In addition, the Agenda calls for teachers to use diverse instructional strategies such as individual or small group work as well as large group work. Furthermore, students themselves recognize that these suggestions would enable them to be less nervous about math. Referring to the previously mentioned sample, 37% of those believed to have math anxiety tendencies expressed a desire for individual help while 42% thought working together would be an advantage. Twenty-seven percent expressed a desire for some credit even when their final answer was not correct. Fear of being humiliated was also reflected by some who believed they would be less nervous: if the teacher weren't so impatient when they did not understand (15%); if they didn't think their questions were dumb (35%); or if they were not called on during class (35%). Additional comments were — I would be less nervous about math if "I could go to a special class to get help"; "I could do it on my own"; "I could understand"; "when I make silly mistakes people won't laugh"; "we had more time to do math"; "students wouldn't laugh and think you were dumb"; "I could have help without feeling like a jerk"; "the teacher would go slower and explain in a simple form." Indeed it seems that students do have a need for a supportive and positive atmosphere in order to learn.

We need to think about our own classrooms and teaching behaviors. Do we do things which produce anxiety? While teaching a lesson it is difficult to decide whether or not what is said or what happens is affecting students negatively. If we are concerned about the classroom atmosphere, then an effort to determine what has happened during any given lesson must be made. One way to accomplish this is to tape the lesson as a means of afterwards checking for negative comments or other anxiety-producing behaviors. Another possibility is to ask the principal, supervisor, or a colleague to observe a lesson, focusing his/her attention specifically on statements or actions which might cause students to have adverse reactions. Either way, the use of a systematic observation instrument will help make the evaluation an objective one. Such an instrument can be used to separate the teacher's verbal and nonverbal be-

haviors into categories which produce or reduce anxiety. This provides a systematic means for recording the classroom atmosphere as it is controlled by the teacher. Mathematics teachers can create their own instruments or use the one offered in Appendix A of this article together with the coding and summary sheet which can provide a picture of anxiety-producing or reducing behaviors at a glance.

Another obligation we have in our fight against math anxiety is to know our students. Do they come to us having had negative experiences in mathematics classes to the extent that they have developed a distaste for and avoidance of mathematics? Since the situation differs from school to school and class to class, what is true on a national average is not necessarily true in our individual classes. For example, much of the literature addresses the anxiety which females, not males, have toward math. However, the school involved in this study does not fit this mold. Of the 52 with math anxiety tendencies, 28 were males and 24 were females, each representing 16% of their respective genders in the sample.

For us to assume automatically that female students exhibit math anxiety more than males is erroneous. One way to determine if learners possess these tendencies is to administer a questionnaire, preferably at the beginning of the year, asking the students how they feel about mathematics. (Refer to appendix B for a sample survey.) Although results will not necessarily be conclusive, tendencies should emerge.

Another possibility is to be aware of the behaviors which math-anxious students display. According to Kogelman and Warren (1978) signs of anxiety are expressed by students who are missing books, who twist shade cords, who are inattentive, who claim they can't do it, who panic, or who don't ask questions. Once again these suggestions are consistent with the responses of the students in the sample. For example, 46% claim they stare in order to avoid doing math problems. Others ask to go somewhere (21%), write a note (29%), or say they don't understand (31%). Forty percent expect not to understand what is explained in math class. A word of caution is needed here. Do not conclude that every person seen staring has math anxiety. These behaviors are indicators of anxiety, but they are not absolutes. Many students who are comfortable doing math said they do the same things to avoid doing their work. It seems that we need to observe our students and talk to them to determine if they are math-anxious — but let's not be overly "anxious" to come to that conclusion.

However, if we do determine that some students seem to have anxiety tendencies, our responsibility is to help overcome them before long-lasting damage is done. Some methods used by the university math clinics could be tried with secondary students. Ask students to keep a diary of their thoughts and emotional reactions as they work on homework. They should observe what math they do and what math they

avoid. While talking about their feelings, they should be encouraged to recall a positive learning experience they have had in math as well as a negative one (Tobias, 1979). We must try to convince students that their own psychological stress is what gets in their way. This "math stage-fright" can be reduced or controlled so they can get down to learning mathematics. The key is to stop resisting.

Since teachers' schedules might prohibit this approach, perhaps in some schools the guidance department could become involved. If neither is possible, then a concentrated effort by the teacher to create a supportive classroom atmosphere will have to suffice. The teacher should always be willing to listen, be patient, accept educated guesses, stress the importance of thought process, and provide new, positive math experiences, encouraging students to try (Morris, 1981). We should let our students know that we, the teachers, have to persist also in solving problems. Surprising as it may appear to them, we do not always know the answers immediately. Tobias (1979) suggests showing classes our scratch paper to emphasize this point. Finally, we should help students recognize their feelings and realize that such emotions are not unusual (Kogelman, 1978).

In conclusion, there are many students who appear confident and happy when they are with friends in school corridors, yet experience stress, frustration and anxiety in the mathematics classroom. If we, the math educators, are unaware of the problem, it is highly probable that these same students may continue to have negative reactions to mathematics and avoid taking courses beyond the basic requirements. Subsequently, this will limit the number of occupations and college majors open to them as well as allow unnecessary frustration in dealing with everyday math-related tasks to develop.

If we are concerned about all of our students, we cannot ignore the problem of math anxiety. Symptoms are noticeable in secondary students and seem to be the result of negative experiences in mathematics classes. Some have these unfortunate experiences in their elementary school days and come to secondary teachers with a negative reaction toward mathematics. We need to be aware of the problem, know who seems to have it, and control our teaching behaviors by providing a classroom in which anxiety is reduced, not produced.

To achieve these goals, the following steps are suggested:

1. Survey classes to determine what students, if any, have tendencies toward math anxiety.
2. Be aware of behaviors which may indicate avoidance or anxiety. Notice when students repeatedly display such symptoms.
3. If math-anxious students are identified and if time permits, interview these students, have them keep diaries, discuss their feelings toward and experiences in mathematics. Try to get them to stop resisting math and work at learning it instead. When deemed

desirable, request the assistance of the guidance department.

4. Provide a positive, supportive and relaxed atmosphere in the classroom. Intuition and thought process should be emphasized and accepted in an effort to build students' confidence in their mathematical ability.
5. Provide some practical applications and make students aware of the occupations which use mathematics (Saunders, 1980). Inform students of the consequences for those who avoid math.

We, the mathematics educators, can work toward solving the problem of math anxiety. The solutions are in our hands. They require no extra materials, no financial backing, no schedule changes, no special permissions from administrators. There are no forms to complete. We can accomplish at least a reduction in, if not an elimination of, math anxiety by controlling our verbal and nonverbal teaching behaviors and by creating a supportive classroom environment where intuition and the thought process involved in problem solving are of utmost importance.

APPENDIX A
Operational Framework for the Analysis of Teacher Behaviors Which Reduce or Produce Students' Math Anxiety

Categories

1. Statements which criticize or ridicule
2. Statements, questions, or directives which cause student discomfort
3. Statements or actions which show impatience or frustration
4. Statements or questions which elicit responses which must meet the teacher's arbitrary standards
5. Statements which are neutral
6. Statements which are self-referenced but are student supportive
7. Statements which praise students
8. Statements which use student contributions
9. Statements which encourage, support, and recognize students' intuition and insight

Category Definitions and Examples

All categories apply to teacher verbal behaviors. Nonverbal behaviors are not recorded except for category three which also includes teacher actions showing impatience. The categories form a continuum from high anxiety producing behaviors to high anxiety reducing behaviors. Categories one through four comprise the anxiety producing zone while six through nine form the anxiety reducing zone. These two zones are separated by category five, the neutral zone.

1. *Statements Which Criticize or Ridicule*

These statements are intended to let the student know that the teacher does not approve of a question or response. This is more than a simple "no" to an incorrect answer. The intent is to convey strong disapproval, almost disgust, to the student for his/her totally unacceptable statement or question. These teacher comments may be sarcastic or humorous, but they belittle the student for his/her math knowledge or lack of it. It is very clear to whom the remark is directed. Often the student's name, you, or your is used.

Examples: "May I assume that you know what a fraction is?"
"You must not be thinking at all."
"I don't see how you even got that answer."
"That's a dumb question."
"Have you considered dropping this course?"
"You think like a girl."
"Math certainly isn't your strong point."
"You'll never get it that way."

2. *Statements, Questions, or Directives Which Cause the Student Discomfort*

These teacher behaviors cause the student discomfort or frustration because they isolate him/her or imply deficiency and underperformance. Very often they compare one student to another. Also included in this category are phrases which cause the student to think he/she is inferior because the material is not clear to him/her.

Examples: "Who isn't finished?"
"Who didn't get that right?"
"John, go to the board and do this problem."
"I want the first person in each row to go to the board and we'll see who can finish #5 first."
"Who doesn't have his/her homework finished?"
"It follows clearly that . . ."
"Obviously, . . ."
"As you can plainly see, . . ."

3. *Statements or Actions Which Show Impatience or Frustration*

The dominant intent of these statements or actions is to indicate to the student that the teacher has run out of patience and is becoming frustrated. These statements do not criticize a student, but because of when they occur, it is possible that certain students could think it is their fault and even feel guilty about causing the teacher to be frustrated.

Examples: "I just explained that!"
"For the fifth time, . . ."
"I repeat once again, . . ."
"I guess I'll have to say it again."
"I give up."

Deep sighing
Frowning
Pacing
Looking at watch frequently
Thowing hands in the air

4. *Statements or Questions Which Elicit Responses That Must Meet the Teacher's Arbitrary Standards*

These statements, questions, or directives imply that the student must give a predictable answer or do a problem in a set way. They do not allow for estimating, intuition, nor work which is done partially correctly. The teacher insists on a single method for doing the problem and guides the discussion in that direction. (Student responses which do not adhere to the set way may elicit criticism or ridicule from the teacher.)

Examples: "Next, you must subtract what from both sides?"
"Your way may get the correct answer, but how did I say it had to be done?"
"John, what is the only thing that I'll let you do next in solving the problem?"
"If you want credit for your work, what steps must you follow in solving this problem?"
"You must always put all of the given first."

5. *Statements Which Are Neutral*

Statements which are managerial or give information about subject content are included in this category. Questions which do not imply that the answer must be approached in the teacher's set way are considered to be neutral. However, the answers are not open-ended, but are factual, such as a definition. Single words or phrases which are used as punctuation or which are habitual should be coded in this category.

Examples: "Yesterday we talked about . . ."
"Today we are going to consider . . ."
"Be quiet."
"Sit down, Bill."
"I'd like you to begin now."
"What is the definition of . . . ?"
"What is the quadratic formula?"
"When would you use completing the square?"

6. *Statements Which Are Self-Referenced but Are Student Supportive*

The intent of self-referenced statements is to present the teacher's feelings and problems encountered in doing math. Comments imply that mathematicians are not born, that mathematics can be difficult even for the teacher who has trouble at times and has to work hard to solve problems. These statements usually include I, my, or me.

Examples: "I don't know."
"Let me think about your question and get back to it tomorrow."
"I had trouble with #7, too."
"I'll have to think about that."
"Let me show you my scratch work for #15."

7. *Statements Which Praise Students*

The dominant intent of these statements is to praise the student for a correct answer. They consist of one word or short sentences which are used basically to reinforce short, factual answers. They do not contain content, instruction or explanation.

Examples: "Good."
"Yes, that's right."
"Uh-huh."
"Good job."
"Correct."
"OK." (in the sense that the answer is correct)

8. *Statements Which Use Student Contributions*

These teacher behaviors are in response to a student initiated question or comment. They include further explanation or more examples which are used because of the student's remark. The dominant intent is to indicate to the students that the teacher is taking time to listen to their problems in understanding and is trying to clarify the concepts for them.

Examples: "Here is another example of . . ."
"Let me say that again."
"Another approach to the same problem is . . ."
"Good idea. I'll prepare some more examples tonight and we'll practice again tomorrow."
"Can anyone help me to explain this?"
"That's a good question."
"I'm glad you brought that up. I'll try to make that clear."

9. *Statements Which Encourage, Support, and Recognize Students' Intuition and Insight*

Teacher behaviors are intended to encourage students to estimate and to use their intuition. Risks are perceived as acceptable and desirable. Special attention to recognizing student understanding (when the student has indeed internalized a concept, has found intrinsic value, has seen the light) is given. These comments are more than simple verification of a right answer coded in #7.

Examples: "How great will the answer be approximately?"
"Can you guess a rule for this?"

"That's great that you can reason that way."
"Did you notice how Sue put lots of ideas together in solving that problem?"
"What do you think might work in solving this problem?"
"Your comments really show that you understand that."
"That's an approach I hadn't thought of."
"I like your way better than mine."

BEHAVIORS WHICH REDUCE OR PRODUCE MATH ANXIETY
CODING AND SUMMARY SHEET

Teacher _____ Grade Level _____
Observer _____ Period _____
Date _____ Subject _____
Lesson _____

ZONES: 5 | 10 | 15 | 20 | 25 | 30 | 35 | 40 | 45 | TOTAL

AREA OF ANXIETY INDUCED AVOIDANCE 6
NEUTRALS 5
APPROACH AVOIDANCE 4
ANXIETY INDUCED AVOIDANCE 3
ANXIETY INDUCED APPROACH 2
AREA OF ANXIETY 1

TOTAL TALLIES

COMMENTS

Summary By Zone

ZONE	TOTAL
ANXIETY REDUCING (6 + 7 + 8 + 9)	
NEUTRAL	
ANXIETY PRODUCING (1 + 2 + 3 + 4)	

APPENDIX B

Name _____ Course _____

This is to find out how you f[illegible] about mathematics. Please read each question carefully and answer it honestly.

1. Whenever it is time to do math, I feel
   (check as many as apply)

_____helpless	_____fearful
_____tense	_____happy
_____glad	_____okay
_____bored	_____unable to cope

2. After I have taken all the math I need to graduate
   (please check one)
   _____I will take more math courses.
   _____I will never take math again.

3. When I go to math class, I know
   (please check one)
   _____I will always understand.
   _____I will usually understand.
   _____I probably will not understand.
   _____I will never understand.

*For each of the following, please circle the number that best describes your feelings:

	Disagree Agree
1. I hate math class.	1 2 3 4 5
2. Math isn't so tough now. But it's going to get really hard very soon.	1 2 3 4 5
3. I tune out in math class. I don't mean to do it — it just happens.	1 2 3 4 5
4. I really don't like asking my math teacher questions.	1 2 3 4 5
5. I panic when called on in math class.	1 2 3 4 5
6. I worry much more about math tests than about any other kind.	1 2 3 4 5
7. I can't wait until I'm old enough to quit math.	1 2 3 4 5

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IS MATHEMATICS ACHIEVEMENT INCREASING OR DECREASING IN PENNSYLVANIA? THE ANSWER IS "YES."

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Headlines read: "SAT Scores Are Going Down," and the "Nation Is at Risk." What is happening to the mathematics skills of students in Pennsylvania? To answer that question, data from the Pennsylvania statewide assessment program — Educational Quality Assessment (EQA) for the years 1978 through 1983, have been analyzed.

EQA assesses grades 5, 8, and 11 in a variety of areas, both cognitive and attitudinal. The mathematics component is very basic. There are sixty mathematics items at each grade and even at grade 11 much of the test is computation. There are no algebraic equations (other than simple linear equations), no geometric proofs and no trigonometric functions on the test. There are, however, multi-step problems and statement problems similar to those typically found in the curriculum.

About 40,000 students at each of the three grade levels are administered the test annually. Every student, other than special education students, in the school district involved, takes the test. This represents annually about one third of the districts in the state, plus one third of the schools in Philadelphia and Pittsburgh.

With such a large statistical sample of schools and students, little change in scores would be expected from one year to the next. There might, however, be small changes within clusters of items (e.g., fractions or measurement) which are not large enough to alter the mean mathematics score, thus giving the appearance of "no difference" in overall mathematics achievement.

A better analysis, then, would look at each of the items on the test, and each of the clusters (major topics) of items on the test. Because of small fluctations of the data from year to year making it difficult to clearly establish a trend in scores from 1978 through 1983, correlation coefficients were calculated between the percent of students correctly responding to an item and the year tested. The question to be answered is "on a given item is the percent of students correctly answering the item increasing or decreasing over-time?" A positive correlation implies that achievement is increasing and a negative correlation implies that achievement is decreasing (FIGURE 1).

EXAMPLE OF PERCENT OF STUDENTS CORRECTLY ANSWERING AN ITEM

Year	Item #1	Item #2	Item #3
1978	67	32	39
1979	68	33	41
1980	67	33	41
1981	66	34	40
1982	65	34	41
1983	64	34	41
Correlation	-.91*	+ .92*	.58

*Statistically significant at .01.

Figure 1

Of the three items shown in Figure 1, Item 1 shows a statistically significant decrease from 1978 to 1983, Item 2 shows a statistically significant increase and Item 3 shows an increase but it is not statistically significant. Obviously, with only six pairs of numbers to correlate, a very large correlation coefficient is necessary for statistical significance.

Figure 2 displays the total number of items that produced a positive correlation between the proportion of students correctly responding to the item and the year tested (an increase in achievement across time) and the total number of items that produced a negative correlation between the proportion of students correctly responding to the item and the year tested (a decrease in achievement across time).

NUMBER OF ITEMS ON THE 60 ITEM TEST SHOWING A POSITIVE OR NEGATIVE CORRELATION BETWEEN THE PROPORTION OF STUDENTS CORRECTLY RESPONDING AND THE YEAR TESTED

	Positive Correlation (Achievement Increasing)	Negative Correlation (Achievement Decreasing)	Significantly Positive	Significantly Negative
Grade 5	39	21	2	1
Grade 8	37	23	4	3
Grade 11	12	48	2	11

Figure 2

The trend is clear. At grades five and eight, achievement is increasing on the EQA mathematics test, while at grade eleven, achievement is decreasing.

Figure 3 contains a breakdown of the items by topic. A plus sign indicates that more items in the cluster had a positive correlation than had a negative correlation. A negative sign represents the reverse situation and a zero means as many were positive as were negative.

	Operations With						
	Whole Nos.	Fractions	Decimals	Geometry	Measurement	Number Patterns	Miscellaneous (Ratio/Percent/ Statistics, Etc.)
Grade 5	+	+	+	+	−	+	0
Grade 8	+	0	+	0	−	−	+
Grade 11	−	−	−	−	−	−	−

Figure 3

Several patterns are evident from Figure 3. First, as expected from Figure 2 data, grade 5 and grade 8 show general increases in achievement while grade 11 decreases in every category. Within the categories, computation is improving while measurement and number patterns are decreasing. The measurement items include conversion of units *within* the system (e.g., pints to quarts and milli-liters to liters) and estimation of geometric quantities (e.g., a quart of water weights about _____ or a dime has a diameter of about _____). Perhaps the reduced emphasis upon the metric system is reflected in lower measurement scores at all three grade levels. Data on the computation (operations with whole numbers, fractions, and decimals) items suggest that computation is improving. The improvement may shortly show up as increased scores at grade eleven.

A phenomenon evidenced in EQA data and also by data from national tests is the lack of improvement on basic skills from grades eight to grade eleven. For example, an item measuring multiplication of fractions goes from 73 percent correct for eighth graders to 75 percent correct on a parallel item at grade eleven. Similarly, addition of fractions goes from 53 percent correct at grade eight to 57 percent at grade eleven. Perhaps, more important than basic skills, however, is problem solving. On the EQA mathematics test at grades five and eight, there has been no change in the proportion of students getting these items correct but for grade eleven, the scores on the problem solving items are decreasing. These findings are consistent with findings from the National Assessment of Educatonal Progress (NAEP).

It is this type of evidence that influenced a recent Pennsylvania Department of Education sponsored task force to outline a new course to meet the revised State Board of Education Chapter 5 curriculum regulations related to increased graduation requirements in mathematics. The new course or "third course" for students not enrolled in the college

preparatory program, is based upon the assumption that if the students can become proficient in fundamental operations with whole numbers, they will be proficient by grade eleven and if they cannot become proficient, then schools should provide them with calculators and teach the mathematics encountered in life and how to deal with that mathematics using a calculator. (Interestingly, the second International Mathematics Study shows that one-third of the eighth grade teachers in the U.S. do not use calculators in class and another one-third indicate calculators are not even allowed in the school. Imagine, two-thirds of the eighth grade classes make no use of calculators!)

Is mathematics achievement increasing or decreasing in Pennsylvania? The answer really is yes! Basic skills are increasing in the elementary and middle school and are decreasing in the high school, while problem solving skills are somewhat unchanged. Geometry, measurement, number patterns and miscellaneous (ratio/percent/statistics, etc.) all appear to need increased attention at the higher grades. It is expected that the new Chapter 5 curriculum regulations, which require more mathematics for graduation, will result in achievement increases at the high school.

WANTED: WOMEN TO FILL THE GAP!

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Literature abounds with studies on why women are avoiding advanced training in mathematics and science. Reasons vary from fear of being unfeminine (Moulton, 1979); fear of failure and fear of success (Frieze, et al., 1978); lack of confidence (Sherman, 1982); poor perception by females as to the usefulness of the subject (Armstrong, 1982); counselor bias in dealing with female students (APA Task Force, 1975); perception of nurturance as their major arena of competence (Chodorow, 1978); sex role identification perceptions developed before entering school (Kraus, 1981); sex stereotyping by the use of sexist materials in the elementary schools (Roe, 1981); fear of breaking with tradition (Lemkeau, 1979); low parental expectations and encouragement (Graham, 1978); to lack of role models (Von Fossen, 1977).

One fact which does emerge from the studies is that there is no one reason which influences women in their career choices. For whatever the reasons, women are not selecting mathematics and science courses. Furthermore, now that we are well into the computer age, new studies are showing that women are avoiding computer training as well as mathematics and science. Three times as many boys as girls attend computer camps across the country and nearly twice as many 17-year-old boys enroll in computer courses as do girls. (Kolata, 1984).

One significant reason given for computer avoidance by women is that they view it as a male domain. The computer's masculine image as a high-tech hunk of equipment discourages some girls from trying their skills in school (Johnson, 1983). With few exceptions, men design video games, write computer software, sell the machines and teach the courses (Kiesler, et al, 1983).

Another reason women avoid taking computer courses is that many of the skills necessary for success in computer programming and advanced computer technology are the same skills necessary for success in mathematics and science. Therefore the same forces which operate to inhibit women from taking mathematics and science will operate to inhibit them from taking computer science.

Women constitute half the work force, yet, their representation in specific job classifications relating to training in mathematics and science has been traditionally low. Because of the computer revolution many new jobs have been created. Unfortunately, women, by early avoidance, will be unable to be represented in the newly created job market.

Business and scientific leaders are concerned that our supply of males, who traditionally become scientists, engineers, technicians, and computer specialists is not large enough to meet the current and future needs of our technological society.

There are those who would say that women could fill the gap in the job market. Skolnick, et al (1982) state:

> It is a fact of American life today that family survival is dependent on the abilities and incomes of all adults. The kinds of mathematical and technical skills we need to care for our own needs, to be creative, and to survive in the job market escalate daily. At the current pace, computer technology may soon be as basic to literacy as reading and writing. As a society, we cannot afford to inhibit the creativity of over half our population. In these times of economic and environmental crisis, the quality and effectiveness of our social solutions depend on the perspectives that women, as well as men, bring to science and technology. (p. 2)

Yet, we are involved in an educational process that prepares the males of our society with mathematics, science and technical skills that are necessary to functioning in today's world but which fails to encourage that same development for females. It is time to change the educational process in order to equalize the sexual population of our labor forces.

Recognizing there is a trend for female avoidance of computers as well as mathematics and science is part of the battle. The next, and hardest part, is doing something about it.

Here are some suggestions:

1. Provide teacher education programs such as the EQUALS program at the University of California, Berkeley. Activities for teachers, counselors, administrators, and parents are designed to attract and retain women in mathematics and sciences. (Kreinberg, 1983)
2. Educators should provide a safe, structured environment to help children clarify their own values, reassuring them it is all right to consider new possibilities, and increase their awareness of how rigidly defined sex roles obstruct opportunities for both sexes. (Skolnick, et al, 1982)
3. Connect mathematics and science to the lives of people. Take it out of abstraction and show how to use it to solve problems of living — growing food, building houses, cities, parks, caring for animals. (Skolnick, et al, 1982)
4. Examine materials used in the schools, particularly in the elementary schools, to see if they are sexist materials. Teachers need to question students about materials used as to whether they think statements or ideas are true only for one sex. Young children should be prompted continually that pictures depicting one sex could just as easily depict the other. (Roe, 1981)

5. Teach girls that work outside the home is a permanent feature of their lives. (Skolnick, et al, 1982)
6. Offer special courses to help cure "math anxiety" which renders females helpless to deal with numbers. (Johnson, 1983)
7. Provide school-aged girls with opportunities to speak with both collegiate undergraduate and graduate women in the sciences. (Kahle, 1983).
8. Make use of networking by putting females in touch with organizations such as "Women and Mathematics Education," whose goal is increased participation of females in the study and pursuit of mathematics-related careers. (Becker & Jacobs, 1983)
9. Provide seed money for promising approaches to increasing the participation of females in mathematics and the sciences. (Kreinberg, 1983)
10. Plan training and retraining programs for counselors which would expose them to information on sex bias, and feedback on their behavior in counseling situations. (Worell, 1980)
11. Encourage teachers to use peer observation and/or video taping in order to code their behavior as they relate to female vs. male students.
12. Encourage use of LOGO, a computer language which can be used by very young children. The assumption is that if a child can write, draw, or make music in ways s/he find interesting, then sex difference in computing may not appear. (Kiesler, et al, 1983)

Women have been avoiding mathematics and science and are now avoiding computer sciences. By allowing them to continue to do so, we perpetuate the myth that women just cannot do well in subjects or careers requiring those subjects, deny half the population their right to reach their fullest potential educationally, and deny women their right to earn high salaries.

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APPLICATIONS IN THE CLASSROOM

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Most mathematics educators are acutely aware of the need to integrate applications into their teaching in order to make the material relevant. How to do this has always been a problem. The examples of applications and "reading problems" in textbooks are usually artificial and often ridiculous. When many of the current practicing teachers were studying mathematics, they were rarely told or shown where and how the mathematics they were learning could be used. Quite often, they didn't care about applications because they loved the mathematics and didn't need pragmatic reasons to study it. As a result, many of us don't *know* where and how much of the mathematics content that we teach is used, and we feel inadequate to explain the physics, or science in general, that relies on the concepts being taught in our mathematics classrooms. Even Euclid faced this problem. When he was asked the question, "But where do I *use* this?" his response was, "Give the boy a coin for he must have something to show for what he has learned." Many of us have been hiding behind that response for years. It's not good enough now. It never was.

In my search for solutions to the problem of not having a pool of realistic applications that could be used in my classroom, I have tried a couple of things. These ideas are described in the following paragraphs with the intent that other teachers who are interested in using and improving on them will do so and, in turn, pass them along to others.

During the 1983-84 school year, my trigonometry and analysis class was studying conic sections at the same time the physics classes were studying the motion of the planets. In relating this coincidence to me one of my students said, almost in awe, "Did you know that planets travel in the paths of ellipses?" If all my students had been taking physics it would have been a dramatic example for all of them. But they don't all take the same science courses and, for some, the moment was lost. It was at this point that I contacted the physics teacher and asked if he would consider video taping some of his experiments so we could bring them into the mathematics classes at strategic times to reinforce what they were learning. He was very cooperative and agreed to this idea.

Generally, we tried to limit the tapes to 15-20 minutes in length so that I would have time to lead in to any given tape and time to follow up in the same class period. We could stop the tapes at any time in order to point out particular concepts that were being used. The construction and use of equal vectors, the need to solve an equation for a particular variable when

several variables were present and several other algebraic manipulations were noted.

Generally things have worked out well but we have run into several difficulties. The biggest problem is lack of adequate time — time to make the tapes, time to review and edit the tapes, and time to show the tapes to other mathematics teachers so they can use them. Having the proper equipment available to perform some of the physics experiments was also a problem. I had the notion that experiments were conducted every week in physics labs and it would be just a case of selecting the most exciting of these experiments to tape. Unfortunately, that was not the case and it was only because of the dedication of our physics teacher that I was able to get some experiments on tape.

We felt that the video tapes would have more impact on our students if they could see that other students were involved in the experiments. Dr. Steve Turns was working with a group of high school students in the Applied Research Laboratory summer intern program at Penn State University, and he agreed to let me tape an experiment entitled "Bernoulli's Wastebasket" that this group of students was doing. It was beautiful! He developed the equations necessary to compute the time required to empty a cylinder of water. In doing this he used differential equations that could be solved by separating variables. The students then filled a large container with water, took measurements and on the basis of their data predicted how long it would take to empty the "basket." Dr. Turns pointed out that this would be quite helpful if you were working with a fluid like maple syrup and had to drain it from containers. Instead of waiting to see how long it would take, you could use the formulas and schedule your time accordingly. I was able to use the tape early in the year in my calculus class when we first talked about differential equations. It gave the class an actual example of the material being used outside a mathematics classroom and, since two of the students in the film were also in my class, it made quite an impression.

As often happens, the idea began to grow. A computer science teacher, another physics teacher, the counselor for the gifted program and I met during the summer and developed a plan to involve more students and ultimately get more tapes and more applications. Our plan was to invite 15 students to work on research projects during the current year. We made a list of "suggested problems" with the idea that they were only suggestions and that the students would be encouraged to expand on these ideas or come up with problems of their own. The problems were to involve mathematical analysis, experiments, the use of audio-visual equipment and computer simulations, if possible. The problems we suggested ranged from Fractals to tea leaves. Several were obtained from a fantastic little book called *The Flying Circus of Physics*, by Gerald Walker.

To help us determine who would participate in this project, we asked

students to submit proposals outlining their problem and what they intended to do with it. After careful review of the submitted proposals, the four of us selected 15 students. Because the projects would involve a great deal of time and effort on the part of the students, we were willing to recognize their contributions. We decided to accept the projects in lieu of a final exam in mathematics, a final project in computer mathematics and some class time and work in physics. Students involved in the project must be taking, or have completed at least two of the following: the advanced placement Pascal course, physics, advanced placement or honors mathematics. The project must be presented to a selected audience some time in May, 1985. As of this writing, the enthusiasm among the students is high and several interesting problems have surfaced.

A criticism recently leveled at high schools is that we don't make our students do research. In a limited manner, we hope this will at least be a start. We are also excited about the interdisciplinary aspect of these projects. As the projects progressed, the English department became involved by offering a technical writing seminar during an activity period and the counselor for the gifted offered a "creative problem solving" seminar.

The disadvantage of the plan is the lack of adequate time. It takes a considerable amount of time to plan the program and, we anticipate, a tremendous amount of time to follow through. Since none of us are "experts" on these problems we will try to find mentors who can advise our students. At this writing the final outcome of the program is unknown. But we have high expectations.

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